Commun. Comput. Phys. Vol. **35**, No. 5, pp. 1418-1444
doi: 10.4208/cicp.OA-2023-0208 May 2024

Application of Continuous Data Assimilation in High-Resolution Ocean Modeling

Adam Larios[1,*], Mark Petersen[2] and Collin Victor[1,2]

[1] *Department of Mathematics, University of Nebraska–Lincoln, Lincoln, NE 68588-0130, USA.*
[2] *Los Alamos National Laboratory, Computational Physics and Methods Group, Los Alamos, NM 87545, USA.*

Received 5 August 2023; Accepted (in revised version) 30 January 2024

Abstract. We demonstrate a formulation of the Azouani-Olson-Titi (AOT) algorithm in the MPAS-Ocean implementation of the primitive equations of the ocean, presenting global ocean simulations with realistic coastlines and bathymetry. We observe an exponentially fast decay in the error before reaching a certain error level, which depends on the terms involved and whether the AOT feedback control term was handled implicitly or explicitly. A wide range of errors was observed for both schemes, with the implicit scheme typically exhibiting lower error levels, depending on the specific physical terms included in the model. Several factors seem to be contributing to this wide range, but the vertical mixing term is demonstrated to be an especially problematic term. This study provides insight into the promises and challenges of adapting the AOT algorithm to the setting of high-resolution, realistic ocean models.

AMS subject classifications: 35Q86, 35Q93, 37N10, 86A05, 34D06, 93C20

Key words: Continuous data assimilation, Azouani-Olson-Titi, oceanography, MPAS-Ocean, primitive equations, nudging.

1 Introduction

In the modeling of turbulent flows, such as ocean dynamics, there are many non-trivial issues that arise. One such issue is the initialization of the model. That is, even if one could evolve the model forward in time exactly, one still would require an initial state to evolve forward from. For many dynamical systems of interest it is infeasible to obtain a complete set of initial data with which to initialize the model. One way to mitigate

*Corresponding author. *Email addresses:* `alarios@unl.edu` (A. Larios), `mpetersen@lanl.gov` (M. Petersen), `collin.victor@tamu.edu` (C. Victor)

this problem is to use a set of techniques, collectively known as data assimilation, to incorporate observational data into the model.

Data assimilation refers to a class of schemes that combine observational data with a physical model to better predict the future state of a physical system. Classical methods of data assimilation include the Kalman filter and its variants, as well as by variational methods such as 3D/4D Var. For a look at classical methods of data assimilation see, e.g., [3, 46] and the references therein. In the past decade, a new data assimilation paradigm known as the Azouani-Olson-Titi (AOT) continuous data assimilation algorithm has developed [4, 5].

The AOT algorithm was first developed in the context of the 1D Allen–Cahn Equation in [5]. It was then given in full generality in [4] for general classes of interpolants in the context of the 2D incompressible Navier–Stokes Equations. Since its inception in 2014, the AOT algorithm has been applied successfully to many other dissipative dynamical systems. This includes the Cahn–Hilliard Equations [16], the 3D primitive equations of the ocean [12,53], the surface quasi-geostrophic equations [38,39], the Kuramoto–Sivashinsky Equations [42, 47, 52], and various regularizations of NSE [1, 32, 41] to name a few. This algorithm has also been used in downstaling a general circulation model of the atmosphere [15]. There have also been numerous studies that utilize the AOT algorithm to recover the solutions to dynamical systems with a variety of restrictions placed on the observational data. This includes observations that include measurement error [6,13,30], incomplete observations of each variable [24,26–29], observations that are discrete or averaged in time [13,30,38,45]. There have also been numerous studies adjusting the gathering and assimilation of observational data, this includes assimilating observational data on a localized patch of the domain [7], using mobile observers to gather data [7, 31, 44], and using nonlinear variations of the feedback control term [11, 17]. Related algorithms which use interpolated or filtered data as in AOT, but insert the results directly have been studied in [14,36,45,50]. Recently there has been work done on modifying this algorithm for use not only in recovering solutions, but in system identification. In particular, the AOT algorithm has been modified to simultaneously recover the true solution of a dynamical system along with the viscosity [9, 23] or the forcing [25, 48].

The AOT algorithm differs from classical methods of data assimilation by introducing a feedback control term at the PDE level. We note that the AOT algorithm appears similar to a form of data assimilation known as nudging, or Newtonian relaxation proposed and studied in [2, 37]. This similarity is superficial, as the AOT algorithm applies a spatial interpolation which has a large impact on, implementation, practical usage, and convergence rates for the algorithm. For an in depth examination of nudging methods (as opposed to AOT methods) see e.g. [40] and the references contained within.

Consider a dynamical system, e.g. the primitive ocean equations, written abstractly as:

$$\frac{d}{dt}u = F(u). \tag{1.1}$$

Note that, in the above equation, we do not know the initial state $u(0) = u_0$ of the flow.