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A New Class of Very-High-Order Finite Difference TENO Schemes with Discontinuity-Resolving Property for Compressible Euler Equations

Tian Liang[1] and Lin Fu[1,2,3,4,*]

[1] *Department of Mechanical and Aerospace Engineering, The Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong.*
[2] *Department of Mathematics, The Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong.*
[3] *HKUST Shenzhen-Hong Kong Collaborative Innovation Research Institute, Futian, Shenzhen, China.*
[4] *Shenzhen Research Institute, The Hong Kong University of Science and Technology, Shenzhen, China.*

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Abstract. In this paper, a new class of very-high-order finite difference TENO schemes with discontinuity-resolving property for compressible Euler equations is proposed. To achieve arbitrary high-order schemes within a unified framework, a novel dual ENO-like stencil selection strategy is proposed based on the candidate stencil arrangement with incremental width. Based on a tailored coupling strategy, the first three small candidate stencils not only provide the reference to judge whether the candidate stencil with a large width is smooth or not but also determine whether the targeted cell is being crossed by genuine discontinuities or not, by combining the stencil smoothness information of several adjacent cells. Without losing generality, the typical seventh- and ninth-order TENO-DR schemes are constructed in detail and a variety of benchmark-test problems, including broadband waves, strong shock and contact discontinuities are studied. Compared to the state-of-the-art TENO schemes, the present schemes exhibit significantly improved high-resolution property and sharp shock-capturing capability, and thus are promising for more complex compressible flow simulations where the excessive numerical dissipation of existing schemes prevents the concerned flow structures from being properly resolved.

AMS subject classifications: 65M06, 35L65, 76J20, 76N15, 76F05

Key words: WENO schemes, TENO schemes, high-order shock-capturing schemes, low-dissipation schemes, compressible gas dynamics.

*Corresponding author. *Email addresses:* `linfu@ust.hk` (L. Fu), `tliangae@connect.ust.hk` (T. Liang)

1 Introduction

For numerically solving the hyperbolic conservation laws, such as the Euler equations of compressible gas dynamics simulation, one long-standing challenge is the development of the high-order and high-resolution numerical methods [1], which must be capable of resolving a broad range of length scales that are commonly orders of magnitude apart in smooth regions and simultaneously capturing the discontinuities. The apparent contradiction between the demand for low numerical dissipation for resolving small-scale flow structures and the requirement for sufficient numerical dissipation for reliable shock-capturing implies that numerical schemes should be capable of adjusting the dissipation property according to the local flow scales.

By using intrinsic limiting procedures, several classical shock-capturing schemes, such as total variation diminishing (TVD) schemes [2, 3], essentially non-oscillatory (ENO) schemes [4], the shock-fitting methods [5], have been presented in the paper to simulate discontinuous flow without numerical oscillations. Different from ENO schemes choosing the smoothest stencil to capture discontinuities from a set of candidate stencils, the family of weighted essentially non-oscillatory (WENO) schemes [6, 7] exploit a series of elaborate weighted coefficients of all candidate stencils to achieve the high-order property and further adjust the local numerical dissipation dynamically. Over the last decades, the development of WENO schemes has been studied by many researchers, and several variants have been developed.

Noticing that the accuracy order of the classical WENO5-JS scheme [7] degenerates near the critical points, where the first- or second-order derivative vanishes, Henrick et al. [8] propose the WENO-M scheme by remapping the WENO5-JS weights at the critical point to satisfy the fifth-order convergence conditions. Then, Borges et al. [9] design a global stencil smoothness indicator, resulting in the WENO-Z scheme, which pushes the nonlinear weights to their optimal values in smooth regions at a much lower computational cost than the WENO-M method. The performance of the WENO-Z scheme is further enhanced in [10–14]. For the purpose of reducing the nonlinear numerical dissipation, a switch function is designed by Hill [15] to control the nonlinear adaptation according to the ratio between the maximum and minimum calculated smoothness indicator. Taylor et al. [16] develop the WENO-RL scheme by directly employing the optimal linear weights when the ratios between each smoothness indicator of small candidate stencils are within a specific range. Therefore, this method matches the spectral characteristics of a nonlinear scheme with its corresponding linear scheme within a certain range of wavenumbers and improves the performance. Hu and Adams [17] further propose an adaptive central-upwind 6th-order WENO scheme (WENO-CU6) by incorporating the contribution from downwind candidate stencil, which enables the upwind scheme to be switched to the central scheme in smooth regions. However, because the background central scheme cannot suppress disturbances from accumulated dispersion errors, spurious waves may occur for flow scales with marginally resolved wavenumbers. Alternative methods attempting to improve the performance of WENO schemes