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Continuation Finite Element Simulation of Second Harmonic Generation in Photonic Crystals

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Abstract. A computational study on the enhancement of the second harmonic generation (SHG) in one-dimensional (1D) photonic crystals is presented. The mathematical model is derived from a nonlinear system of Maxwell's equations, which partly overcomes the shortcoming of some existing models based on the undepleted pump approximation. We designed an iterative scheme coupled with the finite element method which can be applied to simulate the SHG in one dimensional nonlinear photonic band gap structures in our previous work. For the case that the nonlinearity is strong which is desirable to enhance the conversion efficiency, a continuation method is introduced to ensure the convergence of the iterative procedure. The convergence of our method is fast. Numerical experiments also indicate the conversion efficiency of SHG can be significantly enhanced when the frequencies of the fundamental and the second harmonic wave are tuned at the photonic band edges. The maximum total conversion efficiency available reaches more than 50% in all the cases studied.

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1 Introduction

Photonic crystals (PhCs) are artificially fabricated structures in which the index of refraction varies alternatively between high-index regions and low-index regions. Due to its

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periodic variation of the refractive index, a photonic crystal (PhC) exhibits unusual dispersion properties and frequency intervals (i.e., bandgaps) in which propagating Bloch waves do not exist [1,2]. In recent years, nonlinear optical phenomena such as second harmonic generation (SHG) in nonlinear photonic crystals have attracted extensive interest because of the extraordinary capabilities of the photonic crystal structures in controlling the light of visible and infrared wavelengths. It is well known that photonic crystals inhibit propagation of electromagnetic waves within a range of frequencies (photonic band gap). In a number of applications, nonlinear PhC devices offer unique fundamental ways of enhancing a variety of nonlinear optics.

1D photonic band gap (PBG) structures are the modern version of the old interference filters. The structure with index fluctuation contributes to localization of wave across the interfaces of layers with different index of refraction. The nonlinear phenomena in 1D photonic band gap (PBG) structures are governed by the system of nonlinear Maxwell equations. H. Ammari and K. Hamdache have provided rigorous proofs of global existence, uniqueness, and regularity of solutions to a system of nonlinear Maxwell equations in [3]. In previous studies, it has been observed that the enhancement of SH generation in photonic crystals is attributed to the high density modes near the band edges under phase-matching conditions [4, 5]. A general and rigorous theory on SHG in nonlinear multilayered devices can be referred to S. Enoch and H. Akhouayri [6]. They have described a matrix method which can precisely represent the SHG in the depleted pump regime, as well as other nonlinear phenomena such as third-harmonic generation and stimulated Raman scattering.

Recently, an expansion based on the left-to-right (LTR) and right-to-left (RTL) linear modes in 1D nonlinear PhCs was reported in [7–9]. The method analyzes the fully nonlinear system by using a multiple scale expansion approach. Assuming the nonlinear effects should only affect the solution of the linear problem on a length scale much larger than the single element of the structure, the first-order expansion of the electric fields can be expressed as a combination of the LTR modes and RTL modes that depend only on the fast variable and can be calculated by standard matrix transfer techniques. In [8], the nonlinear differential equations of the amplitudes of LTR and RTL modes are solved by using a shooting procedure. This method is efficient in one dimension. Numerical results in [8] showed a number of surprising results. However, this method is based on LTR and RTL modes and multiple scales expansion, it is not straightforward to extend the idea to PBG structures in higher dimensions. In [10, 11], a variational approach that combined the finite element methods and the fixed-point iteration was investigated to study SHG in one dimensional nonlinear optical films, which can be extended to high dimensional structure. But the method fails to produce convergent solution since the iteration scheme may break down when the nonlinearity is very strong.

In this paper, we develop a continuation approach to the combination of finite element methods with a fixed-point iteration algorithm to ensure the convergence of the iterative procedure. We consider the SHG in 1D nonlinear photonic crystal structures in the presence of strong nonlinearity. Numerical results based on this approach indicate