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# Performance of Low-Dissipation Euler Fluxes and Preconditioned LU-SGS at Low Speeds

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**Abstract.** In low speed flow computations, compressible finite-volume solvers are known to a) fail to converge in acceptable time and b) reach unphysical solutions. These problems are known to be cured by A) preconditioning on the time-derivative term, and B) control of numerical dissipation, respectively. There have been several methods of A) and B) proposed separately. However, it is unclear which combination is the most accurate, robust, and efficient for low speed flows. We carried out a comparative study of several well-known or recently-developed low-dissipation Euler fluxes coupled with a preconditioned LU-SGS (Lower-Upper Symmetric Gauss-Seidel) implicit time integration scheme to compute steady flows. Through a series of numerical experiments, accurate, efficient, and robust methods are suggested for low speed flow computations.

**AMS subject classifications**: 65Z05, 76M12, 76N99

**Key words**: All-speed scheme, low-dissipation, preconditioning, LU-SGS.

## 1 Introduction

In recent years, compressible finite-volume methods (FVMs) have been used in a wide spectrum of flow regimes, including low speed flows in which compressibility plays no

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significant role. The application of compressible flow solvers to low speeds has been motivated by the fact that users need only slight modifications to the existing (compressible) codes for computations of such low speed flows, and that this extension has the following potential applications of engineering interests:

1. Analysis of flows involving both low speeds ($M<0.1$) and high speeds ($M\approx 10$ or even 100), e.g., a cavitating flow in a rocket engine [1,2].
2. Aeroacoustic analysis in low speed flows [3].

When applied to low speed flow computations, however, compressible solvers are known to a) fail to converge in acceptable time (stiffness problem), and b) reach unphysical solutions. These problems are known to be cured by A) preconditioning on the time-derivative term so that acoustic wave speed is properly scaled, and B) control of dissipation in numerical fluxes, respectively. There have been several methods of A) [4,5] and B) [6–9] proposed separately. However, it is unclear which combination is the most accurate, robust, and efficient in low speed flows. It is difficult to prove this mathematically because, for instance, the amount of dissipation added to the computation is dependent not only on the adopted methods, but on the computational grid, flow conditions, and so forth. If a combination of methods A) and B) has insufficient dissipation for the given conditions, the calculation will suffer from numerical oscillation/instability, and may eventually diverge. If the method is too dissipative, on the other hand, its accuracy is significantly lost.

Therefore, in the present paper, we pursue an experimental approach by performing a comparative study of different methods of A) along with B) for different grids and different flow conditions of low speeds. We will pay particular attention to several well-known or recently-developed low-dissipation Euler fluxes coupled with a preconditioned LU-SGS (Lower-Upper Symmetric Gauss-Seidel) implicit scheme [10,11] in the framework of steady flows. Similar comparisons have already been conducted by others (in [12], for example), but their discussions were limited to only a few methods/cases and lacked concrete conclusions. In this study, through an extensive series of numerical experiments, accurate, efficient, and robust methods among 14 different approaches will be suggested for low speed flow computations.

The paper is organized as follows: in Section 2, numerical methods and flow conditions adopted here will be described. Then, in Section 3, numerical results and discussions will be presented from a viscous, moderate speed case (Case 1: $M_\infty$=0.5, $Re_\infty$=5,000 in 3.1), inviscid, low speeds cases (Cases 2A-2C: $M_\infty=0.1-0.001$ in 3.2), and a viscous, low speed case (Case 3: $M_\infty$=0.01, $Re_\infty$=2,000 in 3.3). CFL effects will also be discussed in 3.4. These computations will be conducted with global time stepping so that discussions therein could be applied (or at least referenced) to unsteady flow computations with the use of dual-time stepping in which temporal convergence is attained in each time step [4] (not actually covered in this work, though). On the other hand, it is natural to use local time stepping technique if one is interested in steady solutions. Thus, we will address the local time stepping issue as a separate investigation in 3.5. Features of each method will be summarized in 3.6, and Section 4 will conclude the present article.