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An Implementation of MAC Grid-Based IIM-Stokes Solver for Incompressible Two-Phase Flows

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Abstract. In this paper, a novel implementation of immersed interface method combined with Stokes solver on a MAC staggered grid for solving the steady two-fluid Stokes equations with interfaces. The velocity components along the interface are introduced as two augmented variables and the resulting augmented equation is then solved by the GMRES method. The augmented variables and/or the forces are related to the jumps in pressure and the jumps in the derivatives of both pressure and velocity, and are interpolated using cubic splines and are then applied to the fluid through the jump conditions. The Stokes equations are discretized on a staggered Cartesian grid via a second order finite difference method and solved by the conjugate gradient Uzawa-type method. The numerical results show that the overall scheme is second order accurate. The major advantages of the present IIM-Stokes solver are the efficiency and flexibility in terms of types of fluid flow and different boundary conditions. The proposed method avoids solution of the pressure Poisson equation, and comparisons are made to show the advantages of time savings by the present method. The generalized two-phase Stokes solver with correction terms has also been applied to incompressible two-phase Navier-Stokes flow.

AMS subject classifications: 65N06, 35R05, 65M12

Key words: Incompressible two-phase Stokes equations, two-phase Navier-Stokes equations, discontinuous viscosity, singular force, immersed interface method, Uzawa method, front tracking method.

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1 Introduction

Many problems of fluid flows with interfaces between two different fluids have a broad range of natural, science, engineering, and physiological applications. A popular approach for solving such fluid problems on a Cartesian grid is Peskin's immersed boundary method (IBM) [28], which was originally developed to study the fluid dynamics of blood flow in a human heart [27], and was further developed and has been used in a wide variety of applications, particularly applied to biological problems where complex geometries and immersed elastic interfaces are present. Some examples include the deformation of red blood cell in a shear flow [7], swimming of organisms [10], platelet aggregation [11,12], cochlear dynamics [2], biofilm processes [6], wood pulp fiber dynamics [33], and so on. A summary of the development of the immersed boundary method can be found in [28].

An alternative approach is the immersed interface method (IIM) which can capture the solution and its derivative jumps sharply and maintains second-order accuracy via incorporating the known jump conditions into the finite difference approximations near the interface. The IIM was originally proposed by LeVeque and Li [19] for solving elliptic equations, and was later extended to Stokes flow with elastic boundaries or surface tension [18]. The IIM was developed further for the Navier-Stokes equations in [16,17,23,25,38]. The IIM was also used in [4,24,30] for solving the two-dimensional streamfunction-vorticity equations on irregular domains. Xu and Wang [39] have extended the IIM to the 3D Navier-Stokes equation for simulating fluid-solid interaction. Other more applications on the IIM can be found in Li's recent review article [20] or the book by Li and Ito [21] and the references therein.

Recently, Li et al. [22] developed an augmented IIM for incompressible 2D Stokes flows with discontinuous viscosity. However, the method employed a explicit time stepping and a standard (not MAC) grid, so the time step is strictly restricted and the biperiodic boundary condition also has to be assumed. A numerical method for solving the two-fluid Stokes equations with a moving immersed boundary was presented by Layton [15], which uses integral equations to reduce the two-fluid Stokes problem to the single-fluid case. In [35], Tan et al. developed an IIM for the Navier-Stokes equations with discontinuous viscosity across the interface based on the pressure increment projection method. For most biological flows, however, the Reynolds number is pretty low such that Stokes flow simulation is just simply more appropriate. The approach of [35] using Navier-Stokes equation directly to get the steady state solution for such flows is deemed generally impractical and much more expensive. Also using the method in [35] to approach the Stokes flow regime, needs to solve a pressure Poisson equation with discontinuous coefficient subject to Neumann boundary condition, which is a fairly large time-consuming step. On the other hand, to the best of our knowledge, no work on IIM for the steady two-phase Stokes flows involving moving interface with Dirichlet boundary conditions exists in the literature so far. The previously published work on IIM for two-phase Stokes flow with interface is based on solving three Poisson problems