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A Preconditioned Implicit Free-Surface Capture Scheme for Large Density Ratio on Tetrahedral Grids

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Abstract. We present a three dimensional preconditioned implicit free-surface capture scheme on tetrahedral grids. The current scheme improves our recently reported method [10] in several aspects. Specifically, we modified the original eigensystem by applying a preconditioning matrix so that the new eigensystem is virtually independent of density ratio, which is typically large for practical two-phase problems. Further, we replaced the explicit multi-stage Runge-Kutta method by a fully implicit Euler integration scheme for the Navier-Stokes (NS) solver and the Volume of Fluids (VOF) equation is now solved with a second order Crank-Nicolson implicit scheme to reduce the numerical diffusion effect. The preconditioned restarted Generalized Minimal RESidual method (GMRES) is then employed to solve the resulting linear system. The validation studies show that with these modifications, the method has improved stability and accuracy when dealing with large density ratio two-phase problems.

AMS subject classifications: 65E05

Key words: VOF, level set, free surface, unstructured finite volume method, implicit method, restarted GMRES, tetrahedral grid.

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1 Introduction

We have recently reported a novel Coupled Level Set/VOF method for interfacial flow simulations on three dimensional unstructured tetrahedral grids [10]. At each time step, we evolve both the level set function and the volume fraction. The level set function is evolved by solving the level set advection equation using a high resolution characteristic based finite volume method. The volume fraction advection is performed using a bounded compressive Normalised Variable diagram (NVD) scheme. The interface is reconstructed using both the level set and the volume fraction information. In particular, the interface normal vector is calculated from the level set function while the intercepts are determined by enforcing mass conservation based on the volume fraction. The novelty of the method is that we use an analytic method to find the intercepts on tetrahedral grids, which makes interface reconstruction efficient and conserves volume of fluid exactly. The level set function is then reinitialized to the signed distance to the reconstructed interface. Furthermore, the adaptive combination of high resolution discretization schemes ensures the preservation of the sharpness and shape of the interface while retaining boundedness of the volume fraction field. Since the level set function is continuous, the interface normal vector calculation is straightforward and accurate compared to a classic volume-of-fluid method, while tracking the volume fraction is essential for enforcing mass conservation. The method is also coupled to a well validated finite volume based Navier-Stokes incompressible flow solver (Tetrinke) [11,12]. The code validation presented in [10] shows that the proposed method can conserve the mass very accurately and is able to maintain the sharpness of the interface. The coupling of level set and VOF is not the focus of this paper, the details of which can be found in [10]. In our earlier work [10], the discretized equations are marched forward in time using explicit schemes to reduce overall memory consumption. But this low memory demand comes with the sacrifices of stability and convergence speed. Therefore several convergence accelerating techniques including multigrid and implicit residual smoothing are needed to compensate the performance drop. Our numerical analysis has confirmed its good performance using several classical benchmark problems. However, the approximations and point implicit treatment introduced in that method to achieve low-cost computation can destroy the balance between the left-hand side and right-hand side of the equation and can thus slow the convergence rate when dealing with a stiff system (two-phase flow simulation with large density ratio is a good example). Moreover, such explicit schemes will be subject to CFL restrictions. In some simulations, time step size restriction can become a constraining factor, severely limiting the period of the flow simulation. Wave overtopping simulation in coastal engineering is a typical example of this kind, which requires thousands of waves to be simulated before reasonable statistical results can be readily obtained.

Another key feature of our previously proposed free-surface capture scheme is so called air-phase deactivation technique, which deactivates the gaseous phase computation if the density ratio between the two phases exceeds a specified magnitude. It is well