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# Modified Upwinding Compact Scheme for Shock and Shock Boundary Layer Interaction

Chaoqun Liu*, Ping Lu, Maria Oliveira and Peng Xie

*Department of Mathematics, University of Texas at Arlington, Arlington, TX 76019, USA.*

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**Abstract.** Standard compact scheme and upwinding compact scheme have high order accuracy and high resolution, but cannot capture the shock which is a discontinuity. This work developed a modified upwinding compact scheme which uses an effective shock detector to block compact scheme to cross the shock and a control function to mix the flux with WENO scheme near the shock. The new scheme makes the original compact scheme able to capture the shock sharply and, more importantly, keep high order accuracy and high resolution in the smooth area which is particularly important for shock boundary layer and shock acoustic interactions. Numerical results show the scheme is successful for 2-D Euler and 2-D Navier-Stokes solvers. The examples include 2-D incident shock, 2-D incident shock and boundary layer interaction. The scheme is robust, which does not involve case related parameters.

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**Key words**: Compact scheme, WENO, shock-boundary layer interaction, shock detector.

## Nomenclature

| | | | |
|---|---|---|---|
| $M_\infty$ | Mach number | $Re$ | Reynolds number |
| $P$ | Pressure | $\rho$ | Density |
| $U$ | Velocity | $E$ | Internal energy |
| $f$ | Original function | $\hat{f}$ | Flux at the cell interface |
| $F$ | Original function | $H$ | Primitive function of $\hat{f}$ |
| $IS$ | WENO smoothness | $\omega$ | WENO weights |
| $\Omega$ | Control function | | |
| $T_C$ | Truncation error on coarse grid | $T_F$ | Truncation error on fine grid |
| $MR$ | Multigrid truncation error ratio | $LR$ | Local left and right slope ratio |
| CS | Compact Scheme | MUCS | Modified upwinding compact scheme |

*Corresponding author. *Email addresses:* cliu@uta.edu (C. Liu), ping.lu@mavs.uta.edu (P. Lu), malubamol@gmail.com (M. Oliveira), xiepeng.peng@gmail.com (P. Xie)

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## 1 Introduction

The flow field is in general governed by the Navier-Stokes system which is a system of time dependent partial differential equations. However, for external flows, the viscosity is important largely only in the boundary layers. The main flow can still be considered as inviscid and the governing system can be dominated by the time dependent Euler equations which are hyperbolic. The difficult problem with numerical solution is the shock capturing which can be considered as a discontinuity or mathematical singularity (no classical unique solution and no bounded derivatives). In the shock area, continuity and differentiability of the governing Euler equations are lost and only the weak solution in an integration form can be obtained. The shock can be developed in some cases because the Euler equation is non-linear and hyperbolic. On the other hand, the governing Navier-Stokes system presents parabolic type behavior and is therefore dominated by viscosity or second order derivatives in the boundary layer. One expects that the equation should be solved by a high order compact scheme to get high order accuracy and high resolution. High order of accuracy is critical in resolving small length scales in flow transition and turbulence processes. However, for hyperbolic systems, the analysis already shows the existence of characteristic lines and Riemann invariants. Apparently, the upwind finite difference scheme coincides with the physics for a hyperbolic system. History has shown the great success of upwind technologies. From the point of view of shocks, it makes no sense to use high order compact schemes for shock capturing. High order compact schemes use all gird points on one grid line to calculate the derivative by solving a tri-diagonal or penta-diagonal linear system. However, the shock does not have finite derivatives and downstream quantities cannot cross shock to affect the upstream points. From the point of view of the above statements, upwind scheme is appropriate for the hyperbolic system. Many upwind or bias upwind schemes have achieved great success in capturing the shocks sharply, such as [4,15], MUSCL [21], TVD [5], ENO [6,17,18] and WENO (see [7,12]). All these shock-capturing schemes are based on upwind or bias upwind technology, which is nice for hyperbolic systems, but is not favorable to the N-S system which presents parabolic equation behavior. The small length scale is very important in the flow transition and turbulence process and thus very sensitive to any artificial numerical dissipation. High order compact schemes (see [10,22]) are more appropriate for simulation of flow transition and turbulence because it is central and non-dissipative with high order accuracy and high resolution.

Unfortunately, the shock-boundary layer interaction, which is important to high speed flow, is a mixed type problem which has shock (discontinuity), boundary layer (viscosity), separation, transition, expansion fans, fully developed turbulence, and reattachment. In the case of shock-boundary layer interaction, there are elliptic (parabolic for time dependent problems) areas (separation, transition, turbulence) and hyperbolic areas (main flow, shocks, expansion fans), which makes the accurate numerical simulation extremely difficult if not impossible. We may divide the computational domain into several parts: the elliptic (parabolic for time dependent problems), hyperbolic, and mixed.