Commun. Comput. Phys.
doi: 10.4208/cicp.150510.150511s

Vol. **11**, No. 4, pp. 1397-1414
April 2012

A Novel Dynamic Quadrature Scheme for Solving Boltzmann Equation with Discrete Ordinate and Lattice Boltzmann Methods

C. T. Hsu*, S. W. Chiang and K. F. Sin

Department of Mechanical Engineering, Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong.

Received 15 May 2010; Accepted (in revised version) 15 May 2011

Available online 30 November 2011

Abstract. The Boltzmann equation (BE) for gas flows is a time-dependent nonlinear differential-integral equation in 6 dimensions. The current simplified practice is to linearize the collision integral in BE by the BGK model using Maxwellian equilibrium distribution and to approximate the moment integrals by the discrete ordinate method (DOM) using a finite set of velocity quadrature points. Such simplification reduces the dimensions from 6 to 3, and leads to a set of linearized discrete BEs. The main difficulty of the currently used (conventional) numerical procedures occurs when the mean velocity and the variation of temperature are large that requires an extremely large number of quadrature points. In this paper, a novel dynamic scheme that requires only a small number of quadrature points is proposed. This is achieved by a velocity-coordinate transformation consisting of Galilean translation and thermal normalization so that the transformed velocity space is independent of mean velocity and temperature. This enables the efficient implementation of Gaussian-Hermite quadrature. The velocity quadrature points in the new velocity space are fixed while the correspondent quadrature points in the physical space change from time to time and from position to position. By this dynamic nature in the physical space, this new quadrature scheme is termed as the dynamic quadrature scheme (DQS). The DQS was implemented to the DOM and the lattice Boltzmann method (LBM). These new methods with DQS are therefore termed as the dynamic discrete ordinate method (DDOM) and the dynamic lattice Boltzmann method (DLBM), respectively. The new DDOM and DLBM have been tested and validated with several testing problems. Of the same accuracy in numerical results, the proposed schemes are much faster than the conventional schemes. Furthermore, the new DLBM have effectively removed the incompressible and isothermal restrictions encountered by the conventional LBM.

AMS subject classifications: 35Q20, 76N15

Key words: Dynamics quadrature, dynamic discrete ordinate method, dynamic lattice Boltzmann method, thermal lattice Boltzmann method.

*Corresponding author. *Email addresses:* `mecthsu@ust.hk` (C. T. Hsu), `ben.chiang@family.ust.hk` (S. W. Chiang), `meskf@ ust.hk` (K. F. Sin)

1 Introduction

It has been well established that gas flows can be described by the Boltzmann equation (BE) derived from statistical mechanics based on kinetic theory of molecules. However, the Boltzmann equation is a time-dependent nonlinear differential-integral equation in 6 dimensions whose solution is very complicated, difficult and rare. The currently more simplified approach is to linearize the collision integral in BE by the BGK model with an equilibrium Maxwellian distribution and to approximate the moment integrals by the discrete ordinate method (DOM) [1–4] using a finite set of velocity quadrature points. This reduces the dimensions of BE from 6 to 3, and leads to a finite set of linearized Boltzmann equations which can be solved numerically.

The discrete ordinate method (DOM), that had been used long to solve Boltzmann equation for gas flows, was pioneered by Broadwell [1, 2] who employed a very small set of discrete velocities but was able to produce shocks. With the increase in computing power of computer in the last two decades, the DOM has attracted great attention for solving the Boltzmann equation using a large number of discrete velocities. All these early treatments made use of discretization with quadrature points in the velocity space to construct a discrete collision mechanism on the each grid node [5, 6]. A quadrature using fixed velocity points in real physical space to approximate integrals could not be implemented efficiently for obtaining hydrodynamic moments, particularly for high Mach number flows. The difficulty stems from the fact that the accurate integration of Maxwellian distribution depends highly on the temperature and the mean velocity. This requires the use of large number of quadrature points to maintain the integration accuracy when Mach number is high. As a result, huge computational resources are required to capture the flow characteristics.

In this paper, a dynamic quadrature scheme (DQS) for DOM that requires only small quadrature points to approximate accurately the moments of velocity distribution function is proposed. This is achieved through a velocity-coordinate transformation featured with Galilean translation and thermal normalization. The transformation renders the normalized Maxwellian equilibrium distribution with directional isotropy and spatial homogeneity, which enable the accurate and efficient implementation of the Gaussian-Hermite quadrature. The velocity quadrature points in the transformed velocity space are fixed while the correspondent velocity quadrature points in the physical space change from time to time and from position to position. By this dynamic nature in the physical space, we term this new scheme as the dynamic quadrature scheme (DQS). A discrete ordinate method (DOM) with the DQS is then termed as the dynamic discrete ordinate method (DDOM).

Lattice Boltzmann method (LBM), which had been developed for decades, is also a popular and powerful numerical tool to solve the Boltzmann equation for gas flows [7–10]. The LBM also uses discrete velocity set as the DOM used, except that discrete velocities in LBM are specifically assigned to ensure that a particle leaves one lattice node always resides on another lattice node. Hence the LBM can be regarded as a subset of