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Review Article

A Review of Residual Distribution Schemes for Hyperbolic and Parabolic Problems: The July 2010 State of the Art

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Abstract. We describe and review non oscillatory residual distribution schemes that are rather natural extension of high order finite volume schemes when a special emphasis is put on the structure of the computational stencil. We provide their connections with standard stabilized finite element and discontinuous Galerkin schemes, show that their are really non oscillatory. We also discuss the extension to these methods to parabolic problems. We also draw some research perspectives.

AMS subject classifications: 65M60, 76M30, 76M12

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1 Introduction

The numerical simulation of compressible flow problems, or more generally speaking, of partial differential equations (PDEs) of hyperbolic nature, has been the topic of a huge literature since the seminal work of von Neuman in the 40's. Among the "hot" topics of the field has been, since the works of Lax, Wendroff, Godunov, McCormack, van Leer, Roe, Harten, Yee and Osher, to give a few names, the development of robust, parameter free and accurate schemes. Among the most successful methods one may quote the van Leer's MUSCL method [41] and modified flux approach of Roe. These techniques are only second order accurate. The accuracy can be improved via the ENO/WENO methods by Harten, Shu and others.

The emergence of modern parallels computers, another concern has emerged: what about accuracy and efficiency? Indeed, it is now important to develop robust algorithms that scale correctly on parallel architecture. This can be achieved more or less easily if the stencil of the numerical scheme is as compact as possible. Good candidates are the schemes relying on finite element technology, such as the Discontinuous Galerkin (DG) methods [14] or the stabilized continuous finite element(CFE) methods [21, 22]. In these methods, the numerical stencil is the most possible compact one.

In these notes, we discuss in some details of another class of numerical schemes, the so-called Residual Distribution schemes (RD for short), also denoted by Fluctuation Splitting schemes. The history of these schemes can be traced back to the work of P.L. Roe [35] and even his famous 1981 paper [36] where he does not define a finite volume scheme but a true residual distribution scheme. Indeed, the first RD scheme ever was probably presented by Ni [25]. The idea was to construct a scheme with the most compact computational stencil that can ensure second order accuracy. This scheme had some similarities with the Lax Wendroff one.

If these RD share many similarities with more established schemes such as the SUPG scheme by Hughes and coworkers [19–21], the driving idea is (i) to introduce the upwinding concept, (ii) to manage such that a provable or a practical maximum principle is achieved without any parameter to tune. In our opinion, (ii) is the most important feature.

In Roe's paper and the first RD papers, the main idea was to introduce upwinding into the numerical formulation of the problems, coupled in a very clever way, with a technique to reach second order accuracy for steady problem. This has been presented in a series of papers and VKI reports, see e.g. [15,16,28,39,40]. Two schemes had emerged at the time: the N scheme by Roe and the PSI scheme by R. Struijs, see [16]. The first one is probably the optimal first order strategy for scalar problems using triangular meshes, the second one the best second order scheme on these type of meshes, for steady problems again. When dealing with systems or non triangular meshes, the situation became more complex, and it appeared that the upwinding concept had to be relaxed a bit.

Since the early days, many contributions have been given. Among the issues, two are more difficult because they do not cast a priori naturally in the original RD framework.