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Stable and Efficient Modeling of Anelastic Attenuation in Seismic Wave Propagation

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Abstract. We develop a stable finite difference approximation of the three-dimensional viscoelastic wave equation. The material model is a super-imposition of N standard linear solid mechanisms, which commonly is used in seismology to model a material with constant quality factor Q. The proposed scheme discretizes the governing equations in second order displacement formulation using $3N$ memory variables, making it significantly more memory efficient than the commonly used first order velocity-stress formulation. The new scheme is a generalization of our energy conserving finite difference scheme for the elastic wave equation in second order formulation [SIAM J. Numer. Anal., 45 (2007), pp. 1902–1936]. Our main result is a proof that the proposed discretization is energy stable, even in the case of variable material properties. The proof relies on the summation-by-parts property of the discretization. The new scheme is implemented with grid refinement with hanging nodes on the interface. Numerical experiments verify the accuracy and stability of the new scheme. Semi-analytical solutions for a half-space problem and the LOH.3 layer over half-space problem are used to demonstrate how the number of viscoelastic mechanisms and the grid resolution influence the accuracy. We find that three standard linear solid mechanisms usually are sufficient to make the modeling error smaller than the discretization error.

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1 Introduction

Dissipative mechanisms in the earth lead to anelastic attenuation of seismic waves [1]. This attenuation is commonly modeled by describing the earth as a viscoelastic constant-Q absorption band solid, meaning that the material has a quality factor Q, which is independent of frequency. Such material behavior can be approximated in the time-domain by superimposing n standard linear solid (SLS) mechanisms [4].

In this article we develop a stable finite difference approximation of the three-dimensional viscoelastic wave equation with an n-SLS material model. The proposed scheme discretizes the governing equations in second order displacement formulation using $3n$ memory variables, making it significantly more memory efficient than the commonly used first order velocity-stress formulation. The discretization is a generalization of our summation-by-parts finite difference discretization of the elastic wave equation [19, 20, 22]. Our main result is a proof that the proposed discretization is energy stable, even in the case of variable material properties.

There is a substantial number of papers on anelastic attenuation in the literature on seismic wave propagation. Liu et al. [16] showed that the constant-Q material behavior can be approximated by superimposing n standard linear solid (SLS) mechanisms. Day and Minister [7] introduced a rational approximation of the viscoelastic modulus, which enabled realistic attenuation to be introduced in a time-domain seismic wave simulation. Emmerich and Korn [8] pointed out that the rational approximation of the viscoelastic modulus represents the rheological model of a generalized Maxwell body. They devised a least-squares technique of optimizing the coefficients in the rational approximation, which gave a significantly improved approximation of the constant-Q behavior. Moczo and Kristek [18] showed that the generalized Maxwell body used by Emmerich and Korn is equivalent to superimposing n SLS mechanisms. More recently, Savage et al. [23] found that a SLS with $n=3$ mechanisms gives a close to constant Q-value over 1.7 decades in frequency, and illustrated how a higher number of mechanisms allows the frequency band to be made wider. Komatitisch et al. [13] and Käser et al. [12] reported very accurate results for the LOH.3 test problem [5], using three-dimensional time-domain simulations with $n=3$ or $n=4$ mechanisms. Bielak et al. [3] proposed a memory efficient approach based on the rheological model of a Kelvin-Voigt body in parallel with two Maxwell bodies, and reports an almost constant Q-value over two decades in frequency.

Large computational resources are often required for including realistic viscoelasticity in three-dimensional seismic wave simulations. The reason is that the n-SLS viscoelastic model requires a number of so called memory variables to be evolved together with the primary dependent variables (velocities and stresses, or displacements). Each memory variable adds an extra differential equation into the system that governs seismic wave propagation, and the numbers of extra variables and equations are proportional to n.

In the first order velocity-stress formulation, which commonly is used in seismic applications [10, 12, 15, 24], the isotropic elastic wave equation is a system of nine partial differential equations (PDEs) that govern the three components of the velocity and the