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Improving the High Order Spectral Volume Formulation Using a Diffusion Regulator

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Abstract. The concept of diffusion regulation (DR) was originally proposed by Jaisankar for traditional second order finite volume Euler solvers. This was used to decrease the inherent dissipation associated with using approximate Riemann solvers. In this paper, the above concept is extended to the high order spectral volume (SV) method. The DR formulation was used in conjunction with the Rusanov flux to handle the inviscid flux terms. Numerical experiments were conducted to compare and contrast the original and the DR formulations. These experiments demonstrated (i) retention of high order accuracy for the new formulation, (ii) higher fidelity of the DR formulation, when compared to the original scheme for all orders and (iii) straightforward extension to Navier Stokes equations, since the DR does not interfere with the discretization of the viscous fluxes. In general, the 2D numerical results are very promising and indicate that the approach has a great potential for 3D flow problems.

AMS subject classifications: 65

Key words: Diffusion regulation, spectral volume, high-order, Rusanov flux, Navier Stokes equations.

1 Introduction

The spectral volume (SV) method is a high order method, originally developed by Wang, Liu and their collaborators for hyperbolic conservation laws on unstructured grids [18, 26–30]. The spectral volume method can be viewed as an extension of the Godunov method to higher order by adding more degrees-of-freedom (DOFs) in the form of subcells in each cell (simplex). The simplex is referred to as a spectral volume and the subcells are referred to as control volumes (CV). All the spectral volumes are partitioned in a geometrically similar manner in a simplex, and thus a single reconstruction is obtained. As

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in the finite volume method, the unknowns (or DOFs) are the subcell-averaged solutions. A finite volume procedure is employed to update the DOFs.

The spectral volume method was successfully implemented for 2D Euler [29] and 3D Maxwell equations [18]. Recently Sun et al. [24] implemented the SV method for the Navier Stokes equations using the LDG [6] approach to discretize the viscous fluxes. Kannan and Wang [11] conducted some Fourier analysis for a variety of viscous flux formulations. Kannan implemented the spectral volume method for the Navier Stokes equations using the LDG2 (which is an improvised variant of the LDG approach) [12] and DDG approaches [13]. Even more recently, Kannan extended the spectral volume method to solve the moment models in semiconductor device simulations [8–10]. Other developments include the formulation of a new boundary condition [14] and the implementation for elastohydrodynamic problems [15]. These past studies have demonstrated the efficacy of the spectral volume method for a wide range of engineering applications, and have established its robustness.

In spite of all the above developments, the handling of the inviscid fluxes has undergone minimal changes since the inception of the spectral volume method. Till date, almost all of spectral volume implementations use the Rusanov or the Roe formulation as the approximate Riemann flux. These fluxes utilize an artificial dissipation term (or a matrix) as a straight-forward addition to the averaged flux (central discretization). This simplistic flux evaluation procedure has yielded acceptable results.

In this paper, we borrow ideas from Jaisankar et al. [7] to regulate this artificial dissipation. In particular, we blend this diffusion regulation (aptly called DR), with the Rusanov implementation of the approximate Riemann flux. Numerical experiments (both inviscid and viscous) were conducted to compare and contrast the newly formulated DR and the traditional formulations. The simulations performed with the DR showed dramatic improvements over those employing the traditional approach for 2^{nd}, 3^{rd} and 4^{th} order simulations. Moreover, the DR does not interfere with the viscous flux discretization procedure. Hence it can be used in conjunction, with any viscous flux discretization procedure like the LDG [8, 11], LDG2 [10, 12], penalty [9, 11] or the BR2 [9, 11] formulations.

The paper is organized as follows. In the next section, we review the basics of the SV method. The basics of the DR are discussed in Section 3. Section 4 presents with the different test cases conducted in this study. Finally conclusions from this study are summarized in Section 5.

2 Basics of the spectral volume method

2.1 General formulation

Consider the general conservation equation