

## A New Variational Model with Dual Level Set Functions for Selective Segmentation

Lavdie Rada and Ke Chen\*

*Centre for Mathematical Imaging Techniques (CMIT) and Department of Mathematical Sciences, The University of Liverpool, Peach Street, Liverpool L69 7ZL, United Kingdom.*

Received 19 January 2011; Accepted (in revised version) 21 June 2011

Available online 30 January 2012

---

**Abstract.** In this paper we present a selective segmentation model using a dual level set variational formulation. Our variational model aims to segment all objects with one level set function (global) and the selected object, which is the closest to the geometric constraints (markers), with another level set (local). It is a combination of edge detection, markers distance function and active contour without edges. Experimental results show that our model is more robust than previous work.

**AMS subject classifications:** 62H35, 65N22, 65N55, 74G65, 74G75

**Key words:** Image selective segmentation, level set function, Euler-Lagrange equation, 2D image segmentation, re-initialization.

---

### 1 Introduction

Image segmentation is a central problem among image processing applications. Its aim is to distinguish objects in the image foreground from background and to systematically select specific features out of an image that has many features.

There are different techniques developed for this task such as histogram analysis and thresholding [21, 29, 32], region growing [2, 35], edge detection and active contours [3, 11, 16]. Active contour models are widely used in image segmentation due to their robustness and reliability. These models are formulated as energy minimization problems and can be categorized broadly into edge-based models [6, 16–18], and region-based models [9, 11, 24, 26, 31]. Edge-based models use the edge information (certain form of gradients) guiding the active contours towards the object boundary, and the region based ones make use of image intensities (certain homogeneity) to guide the motion of active contours.

---

\*Corresponding author. *Email addresses:* ladirada@liv.ac.uk (L. Rada), k.chen@liv.ac.uk (K. Chen)

The above image segmentation models are useful for various applications when all features in the whole image are to be segmented. This challenging task is continually tackled by more refined models but is not required in some other applications such as CCTV monitoring of a subject and medical imaging of a particular organ. For the latter applications, the problem becomes selective segmentation.

The main challenge in a selective image segmentation problem is how to differentiate one feature from another, especially when two objects have similar or same intensities. For example, in an artificial image like in Fig. 1 the intensities between the triangle and the rectangle have an extremely small difference and we might be interested in selecting only one of them. Another example is with medical images (CT and MRI) which often have less contrast in image intensities.

Recent work by Gout and Guyader [14] and Badshah-Chen [4] proposed two different variational models for selective segmentation. The Gout-Guyader model [14, 15] is based on edge information of the object while the Badshah-Chen model [4] combines an edge based model with region based information. Both models are useful and can segment a range of images, but there are cases which appear too challenging for either model. The latter model, with the help of region information, improved the former in robustness and segmentation quality in case of noisy images. It should be remarked that for global segmentation, the idea of combining an edge based model with region based information was earlier used in [5, 30]. However we have observed that the particular selective segmentation model of [4] partly based on Chan-Vese model [11], when solved in a time-marching framework, can reproduce the same solution of two piecewise constants as the Chan-Vese model [11]; if this happens, the capability of selectiveness is lost and we obtain a global segmentation which is not needed for our purpose. This problem is illustrated in the top two plots of Fig. 2 (which solves the problem from Fig. 1).

To further improve on the Badshah-Chen work [4] here we will introduce a new model which does two tasks at the same time, one to find the segmentation of all boundaries and the other to focus on the selected object, which is the closest to the geometric constraints (or markers). The first task implements a global segmentation like a region based model while the second one implements a local segmentation using both local edge and local region information. Since each task is characterized by a level set function [1, 7, 10, 17, 23, 25, 28], our model employs two level set functions, namely, the global function  $\phi_G$  and the local function  $\phi_L$ . Once our dual level set variational model is solved, the global level set function  $\phi_G$  will segment the entire domain while the local one will define the desired selective object. However, since a level set function is not unique away from a boundary, a re-initialization might be required. There are two different techniques of re-initialization, either by solving a re-initialization equation [25] or by incorporation of a functional into the minimization problem [18]. Before proceeding, to give an early clue on our new model to be presented shortly, the bottom two plots of Fig. 2 show our new and correctly segmented results.

We remark that our proposed model for local and selective segmentation may be further speeded up by using the numerical methodology as developed by Sethian [27],