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A Class of Hybrid DG/FV Methods for Conservation Laws III: Two-Dimensional Euler Equations

Laiping Zhang[1,2,*], Wei Liu[1], Lixin He[2] and Xiaogang Deng[1,2]

[1] *State Key Laboratory of Aerodynamics, China Aerodynamics Research and Development Center, Mianyang, Sichuan 621000, China.*
[2] *Computational Aerodynamics Institute, China Aerodynamics Research and Development Center, Mianyang, Sichuan 621000, China.*

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Abstract. A concept of "static reconstruction" and "dynamic reconstruction" was introduced for higher-order (third-order or more) numerical methods in our previous work. Based on this concept, a class of hybrid DG/FV methods had been developed for one-dimensional conservation law using a "hybrid reconstruction" approach, and extended to two-dimensional scalar equations on triangular and Cartesian/triangular hybrid grids. In the hybrid DG/FV schemes, the lower-order derivatives of the piecewise polynomial are computed locally in a cell by the traditional DG method (called as "dynamic reconstruction"), while the higher-order derivatives are re-constructed by the "static reconstruction" of the FV method, using the known lower-order derivatives in the cell itself and in its adjacent neighboring cells. In this paper, the hybrid DG/FV schemes are extended to two-dimensional Euler equations on triangular and Cartesian/triangular hybrid grids. Some typical test cases are presented to demonstrate the performance of the hybrid DG/FV methods, including the standard vortex evolution problem with exact solution, isentropic vortex/weak shock wave interaction, subsonic flows past a circular cylinder and a three-element airfoil (30P30N), transonic flow past a NACA0012 airfoil. The accuracy study shows that the hybrid DG/FV method achieves the desired third-order accuracy, and the applications demonstrate that they can capture the flow structure accurately, and can reduce the CPU time and memory requirement greatly than the traditional DG method with the same order of accuracy.

AMS subject classifications: 76G25, 76H05, 76M10, 76M12, 76M25

Key words: Discontinuous Galerkin method, finite volume method, reconstruction, hybrid method.

*Corresponding author. *Email addresses:* zhanglp_cardc@126.com (L. P. Zhang), lw4992@gmail.com (W. Liu), hlx_cardc@126.com (L. X. He), xgdeng@skla.cardc.cn (X. G. Deng)

1 Introduction

While 2nd order methods are dominant in most compressible flow simulations, many types of problems, such as computational aeroacoustics (CAA), vortex-dominant flows and large eddy simulation (LES) of turbulent flows, call for higher order accuracy (third order and more). The main deficiency of widely available, second-order methods for the accurate simulations of the above-mentioned flows is the numerical diffusion and dissipation of vorticity to unacceptable level. Applications of high-order accurate, low-diffusion and low dissipation numerical methods can significantly alleviate this deficiency of the traditional second order methods, improve predictions of vortical and other complex, separated, unsteady flows. Therefore, various high-order methods have been developed in the last two decades, including the essentially non-oscillatory scheme (ENO) [1] and the weighted-ENO scheme (WENO) [2] on structured grids, the discontinuous Galerkin (DG) method [3–7], the ENO and WENO schemes [8–15], the spectral volume (SV) method [16–19], and the spectral difference (SD) method [20–22] on unstructured grids. Interested readers can refer to the comprehensive review articles for high-order methods by Ekaterinaris [23] on structured grids and by Wang [24] on unstructured grids. Because the structured/unstructured hybrid grid technique presents the trend of grid generation technique [25], due to the capability for complex geometries, the high-order methods on unstructured and hybrid (or mixed) grids are paid much more attention in recent years.

As the leader of high-order numerical methods for compressible flow computations in aerospace applications, the DG methods have recently become popular for problems with both complex physics and geometry. The DG method was originally developed by Reed and Hill to solve the neutron transport equation [3]. The development of high-order DG methods for hyperbolic conservation laws was pioneered by Cockburn, Shu and other collaborators in a series of papers on the Runge-Kutta DG (RKDG) method [4–7]. Many other researchers made significant contributions in the development. Refer to [26] for a comprehensive review on the DG method history and literature. The most distinguished feature of the DG methods is the ”compact” property on arbitrary grids.

However, the DG methods have a number of their own weaknesses, concentrating on the huge computational cost (memory requirement and CPU time). The block diagonal matrix requires a storage of $(ndof \times neqs) \times (ndof \times neqs) \times nelems$, where $ndof$ is the number of degrees of freedom (DOFs) for the polynomial, $neqs$ is the number of components in solution vector and $nelems$ is the total cell number of the grid. For example, the storage of this block diagonal matrix alone requires 10,000 words per element for a fourth-order DG scheme in 3D [27]! Indeed, the lack of efficient solver is one of the bottlenecks in the development of the DG methods for solving realistic problems.

Comparing with the traditional 2nd order DG method, the widely available 2nd order finite volume (FV) methods, as well as the finite difference (FD) methods, need smaller memory and computation cost, because they do not have to compute the volume integrals and the additional equations for the DOFs corresponding to the derivatives. In