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A Numerical Study of Quantum Decoherence

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Abstract. The present paper provides a numerical investigation of the decoherence effect induced on a quantum heavy particle by the scattering with a light one. The time dependent two-particle Schrödinger equation is solved by means of a time-splitting method. The damping undergone by the non-diagonal terms of the heavy particle density matrix is estimated numerically, as well as the error in the Joos-Zeh approximation formula.

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1 Introduction

Quantum decoherence is nowadays considered as the key concept in the description of the transition from the quantum to the classical world (see e.g. [6,7,9,19,20,22,24]).

As it is well-known, the axioms of quantum mechanics allow superposition states, namely, normalized sums of admissible wave functions are once more admissible wave functions. It is then possible to construct non-localized states that lack a classical interpretation, for instance, by summing two states localized far apart from each other. The observable mark of such a quantum mechanical superposition state is the presence of interference fringes in the probability distribution associated to the state (see e.g., [11]). We stress that this phenomenon does not have a classical explanation: classically, a probability distribution evolving freely in the phase space of a single particle follows the free

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Liouville equation, so, by linearity, two colliding probability densities sum up without creating an interference pattern.

Nonetheless, at human scale no interference is revealed, so the question arises, on how does the interference pattern disappear. Such a phenomenon is called *decoherence* and its explanation lies in the fact that macroscopic objects undergo a continuous interaction with an external environment (such as air molecules, fields), which causes the loss of the phase relations between the different states in the superposition. Thus, the state of the system becomes a statistical mixture in which the quantum effects are suppressed. In this sense, the system loses its quantum nature and then its state admits a classical interpretation.

Understanding decoherence is important not only in the foundations of quantum mechanics, but also in applied physics. For example, in quantum computation (QC), electron spin resonance (ESR), and nuclear magnetic resonance (NMR) it is of paramount importance to preserve the quantum behaviour, so decoherence is not desired and efforts are made in order to avoid it [27, 29]. On the other hand, in quantum interference effect transistors (QuIET) decoherence is exploited to control the quantum current flow [28]. In such devices, decoherence acts like a switch to modulate the current flow, the device being switched "off" in the completely coherent state and "on" when interference disappears.

We remark that the transition from the quantum to the classical regime due to decoherence is different from the semi-classical limit, where the classical behaviour is recovered exploiting the smallness of Planck's constant. Let us stress three main differences to this regard: first, decoherence requires an open system, i.e. a system that interacts with an environment; second, decoherence acts at the length-scale of the interference pattern, whereas a typical semi-classical procedure consists in evaluating a macroscopic observable on a fast oscillating probability distribution; third, decoherence is a dynamical effect: it grows with time, whereas the semi-classical limit can be performed in the stationary framework too. Furthermore, at least qualitatively, $\hbar$ plays no role in the mechanism of decoherence: nevertheless, from a quantitative point of view, in many models of physical relevance the time-scale of the decoherence owes its shortness to the smallness of $\hbar$ (see e.g. [24]). Even though in this paper $\hbar$ acts as a constant, we keep writing it explicitly in formulas, in view of possible future investigation on a more precise determination of the decoherence rate.

In spite of its recognized relevance, there are still few rigorous results on decoherence, both from the analytical [1–3, 8, 10, 12, 14, 15] and the numerical [5] point of view.

The aim of this paper is to investigate numerically the mechanism of decoherence on the simplest model in which it takes place, namely a heavy particle that scatters a light one. Therefore, the system we monitor is a quantum particle and the environment is another quantum particle, considerably lighter than the first one. The two particles interact with each other via a repulsive potential and, due to the low mass-ratio, the light particle is scattered away while the heavy particle remains almost unperturbed. It turns out that, at leading order in the mass ratio, the only effect of the interaction on the heavy