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REVIEW ARTICLE

Phase-Field Models for Multi-Component Fluid Flows

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Abstract. In this paper, we review the recent development of phase-field models and their numerical methods for multi-component fluid flows with interfacial phenomena. The models consist of a Navier-Stokes system coupled with a multi-component Cahn-Hilliard system through a phase-field dependent surface tension force, variable density and viscosity, and the advection term. The classical infinitely thin boundary of separation between two immiscible fluids is replaced by a transition region of a small but finite width, across which the composition of the mixture changes continuously. A constant level set of the phase-field is used to capture the interface between two immiscible fluids. Phase-field methods are capable of computing topological changes such as splitting and merging, and thus have been applied successfully to multi-component fluid flows involving large interface deformations. Practical applications are provided to illustrate the usefulness of using a phase-field method. Computational results of various experiments show the accuracy and effectiveness of phase-field models.

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Key words: Navier-Stokes, Cahn-Hilliard, multi-component, surface tension, interface dynamics, interface capturing, phase-field model.

Contents

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1 Introduction

Many important industrial problems involve flows with multiple constitutive components. Several examples are the impact of a droplet on a solid surface [53], bubbly and slug flows in a microtube [46], drop coalescence and retraction in viscoelastic fluids [106], and realistic interfaces in computer graphics [1]. Due to inherent nonlinearities, topological changes, and the complexity of dealing with unknown moving interfaces, multiphase flows are challenging to study from mathematical modeling and numerical algorithmic points of view.

There are many ways to characterize moving interfaces. The two main approaches to simulating multiphase and multi-component flows are interface tracking and interface capturing methods. In interface tracking methods (volume-of-fluid [42], front-tracking [40], and immersed boundary [49,50,80,81,96]), Lagrangian particles are used to track the interfaces and are advected by the velocity field. In interface capturing methods such as level-set [24,78,79,85,91,92] and phase-field methods [4,6,13,29,46,47,54,56,74,86,90,102], the interface is implicitly captured by a contour of a particular scalar function.

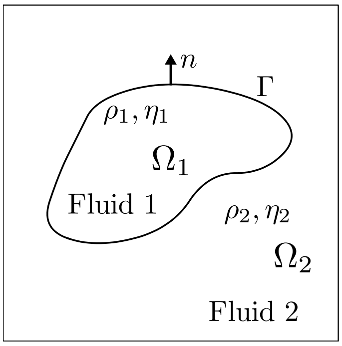

Figure 1: Schematic diagram of a two phase domain.

The governing equations of unsteady, viscous, incompressible, and immiscible two fluid systems in three-dimensional space are the Navier-Stokes equations:

$$\rho_i\Big(\frac{\partial \mathbf{u}_i}{\partial t}+\mathbf{u}_i\cdot\nabla\mathbf{u}_i\Big)=-\nabla p_i+\nabla\cdot[\eta_i(\nabla\mathbf{u}_i+\nabla\mathbf{u}_i^T)]+\rho_i\mathbf{g}, \qquad \text{in } \Omega_i,$$
$$\nabla\cdot\mathbf{u}_i=0, \qquad \text{in } \Omega_i,$$

where $\rho_i(\mathbf{x},t)$ is the density, $\mathbf{u}_i(\mathbf{x},t)=(u_1(\mathbf{x},t),u_2(\mathbf{x},t),u_3(\mathbf{x},t))$ is the velocity, $p_i(\mathbf{x},t)$ is the pressure, and $\eta_i(\mathbf{x},t)$ is the viscosity of fluid $i=1$, 2, the superscript T denotes transpose, and $\mathbf{g}$ is the gravitational force per unit mass. See Fig. 1 for the schematic diagram of a two phase domain. Γ is the interface of the two immiscible fluids and $\mathbf{n}=(n_1,n_2,n_3)$ is the unit normal vector to the interface. On the interface Γ, we have a normal jump condition

$$p_2-p_1=\sigma\kappa+\Big(2\eta n_k\frac{\partial u_k}{\partial n}\Big)_2-\Big(2\eta n_k\frac{\partial u_k}{\partial n}\Big)_1,$$