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An Efficient Sampling Method for Regression-Based Polynomial Chaos Expansion

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Abstract. The polynomial chaos expansion (PCE) is an efficient numerical method for performing a reliability analysis. It relates the output of a nonlinear system with the uncertainty in its input parameters using a multidimensional polynomial approximation (the so-called PCE). Numerically, such an approximation can be obtained by using a regression method with a suitable design of experiments. The cost of this approximation depends on the size of the design of experiments. If the design of experiments is large and the system is modeled with a computationally expensive FEA (Finite Element Analysis) model, the PCE approximation becomes unfeasible. The aim of this work is to propose an algorithm that generates efficiently a design of experiments of a size defined by the user, in order to make the PCE approximation computationally feasible. It is an optimization algorithm that seeks to find the best design of experiments in the D-optimal sense for the PCE. This algorithm is a coupling between genetic algorithms and the Fedorov exchange algorithm. The efficiency of our approach in terms of accuracy and computational time reduction is compared with other existing methods in the case of analytical functions and finite element based functions.

AMS subject classifications: 60H15, 62K20, 62K05

Key words: Polynomial chaos expansion, regression, D-optimal design, Fedorov Algorithm, genetic algorithms.

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1 Introduction

In the recent years, there has been an increasing interest in the simulation of systems with uncertainties. Due to the uncertainties in the input parameters, the response of the mechanical system has a deterministic component and a random component; this random

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component is often ignored in traditional engineering practice. The reliability analysis consists in defining the probability density functions (pdf) of the uncertain input parameters and then propagating them through the mathematical model of the mechanical system in order to characterize the random component of the output.

The polynomial chaos expansion (PCE) method builds a multidimensional polynomial function that approximates the output of the system around its nominal value with respect to the uncertainty of the input parameters. There are two non-intrusive methods to construct the PCE approximation: the projection method and the regression method (see [22] and [23]). Both of them are black box methods that require a set of independent simulations for different values of the input parameters.

The total computational time depends on three factors: the computer resources, the computational time of one finite element analysis (FEA) and the number of simulations needed to build PCE approximation. Usually, the industrial applications require expensive finite element analysis. Hence, if the number of analyses for building the PCE is large, the total computational time becomes unfeasible. Reducing the total computational time by tuning the first two factors is not an easy task. The computer resources are usually limited and reducing the computational time of one FEA means reducing its accuracy. Therefore, one has to reduce the third factor, the number of analyses for the PCE, in order to be able to carry out a reliability analysis of an expensive FEA-based application.

Recently, some new methods have been introduced to reduce the number of independent simulations. The projection method requires running a set of simulations defined over a sparse grid of nodes following some quadrature rules. In [16–19], the reduction of the size of sparse grid is based on the anisotropy of the problem and the contribution of each node of the sparse grid in the overall accuracy of the numerical evaluation.

The regression method requires the definition of a design of experiments depending on the PCE polynomial function. In [1,20,21], a model reduction approach is considered where some terms of the PCE approximation are discarded according to their pertinence in the model. This leads to a reduction of the size of the design of experiments.

The preceding methods prescribe their number simulations without providing to the user a direct control over the total number of simulations to perform the PCE approximation. The contribution of this paper is to propose a sampling algorithm for the regression method where the user can choose the number simulations depending on the available computer resources, the computational time of one FEA and the desired total computational time. The proposed algorithm gives the possibility to carry out a PCE approximation with the fewest number of simulations which is equal to the number of terms in the PCE. Therefore, with this approach it is always possible to carry out a reliability analysis based on an expensive FEA model and a large number of uncertain parameters.

The accuracy of the PCE approximation given by the regression method is proportional to the determinant of the regression matrix. The approach proposed in this paper consists in maximizing this determinant for a given number of samples. This leads to a small sized design of experiments that covers efficiently all the space of the uncertain-