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A Lattice Boltzmann and Immersed Boundary Scheme for Model Blood Flow in Constricted Pipes: Part 2 – Pulsatile Flow

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Abstract. One viable approach to the study of haemodynamics is to numerically model this flow behavior in normal and stenosed arteries. The blood is either treated as Newtonian or non-Newtonian fluid and the flow is assumed to be pulsating, while the arteries can be modeled by constricted tubes with rigid or elastic wall. Such a task involves formulation and development of a numerical method that could at least handle pulsating flow of Newtonian and non-Newtonian fluid through tubes with and without constrictions where the boundary is assumed to be inelastic or elastic. As a first attempt, the present paper explores and develops a time-accurate finite difference lattice Boltzmann method (FDLBM) equipped with an immersed boundary (IB) scheme to simulate pulsating flow in constricted tube with rigid walls at different Reynolds numbers. The unsteady flow simulations using a time-accurate FDLBM/IB numerical scheme is validated against theoretical solutions and other known numerical data. In the process, the performance of the time-accurate FDLBM/IB for a model blood flow problem and the ease with which the no-slip boundary condition can be correctly implemented is successfully demonstrated.

AMS subject classifications: 76Z05, 76M20, 65M06

Key words: Finite difference method, lattice Boltzmann method, immersed boundary method, blood flow, constricted pipe.

1 Introduction

It was pointed out in a companion paper [1] that a viable numerical solver for blood flow modeling should at least be able to correctly simulate certain key features of blood flow. The more important of these are non-Newtonian fluid, incompressible pulsating

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flow that could become erratic, three-dimensional (3-D) flow with complex and changing boundaries, and flow-structure interaction resulting from elastic wall boundary. In an attempt to develop an alternative numerical solver based on the lattice Boltzmann method (LBM), Fu and his co-workers [2–6] used their previously developed finite difference lattice Boltzmann method (FDLBM) as base to further extend the FDLBM to handle complex boundary by incorporating the immersed boundary method [7,8] to their FDLBM; thus giving a combined FDLBM/IB numerical scheme [1] where the no-slip boundary condition is just as easy to implement as any other finite difference scheme used to solve the Navier-Stokes (NS) equations. This extension retains the ease with which the FDLBM can still perform computations using Cartesian grids in problems with complex boundaries. The FDLBM [2] can correctly resolve incompressible flow [3]; replicate Newtonian and non-Newtonian flow in microchannel and microtube accurately [2]; simulate incompressible flow with random variations in viscosity and pressure in channel/pipe [4]; and model 2-D and 3-D buoyant flow [5,6]. Therefore, the FDLBM/IB is not only suitable for flow with complex boundary, but is also appropriate for incompressible Newtonian and non-Newtonian flow with external body force, and with random variation of viscosity and pressure.

With these successes, Fu et al. [1] adopted their FDLBM/IB numerical scheme to study steady Newtonian flow in constricted tubes at different Reynolds numbers. The resulting governing equations in the FDLBM/IB scheme are still linear, so they can be solved locally, explicitly and efficiently using Cartesian grids. The numerical scheme was further used to investigate Newtonian and non-Newtonian fluid flow in stenosed arteries and the effect of viscous stress and resistance on disordered flow patterns resulting from the constriction [1]. Their study brought closer to reality the development of LBM as an alternative to numerically solving the NS equations for blood flow simulation. However, the FDLBM and FDLBM/IB [1–6] developed to-date can only handle steady flow with and without random fluctuations. Therefore, its extension to unsteady flow in an axisymmetric domain would represent another forward step to render the method appropriate for blood flow simulation. The next step in the development of an LBM solver for blood flow modeling is to build on the steady FDLBM/IB experience [1], and extend the methodology to unsteady and pulsating flow. This necessitates the extension of the steady FDLBM and FDLBM/IB algorithms to time-accurate FDLBM and FDLBM/IB schemes.

The present paper reports on just this development. After validating the time-accurate FDLBM and FDLBM/IB scheme, numerical simulations of a modeled blood flow problem, namely that of Newtonian and non-Newtonian fluid flow in constricted tubes, using the time-accurate FDLBM/IB scheme are investigated and discussed.

2 The FDLBM/IB numerical scheme

A schematic representation of the blood flow problem, modeled by an incompressible flow through a rigid tube with a localized constriction, is shown in Fig. 1. In this model,