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Development and Application of a Reduced Order Model for the Control of Self-Sustained Instabilities in Cavity Flows

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Abstract. Flow around a cavity is characterized by a self-sustained mechanism in which the shear layer impinges on the downstream edge of the cavity resulting in a feedback mechanism. Direct Numerical Simulations of the flow at low Reynolds number has been carried out to get pressure and velocity fluctuations, for the case of un-actuated and multi frequency actuation. A Reduced Order Model for the isentropic compressible equations based on the method of Proper Orthogonal Decomposition has been constructed. The model has been extended to include the effect of control. The Reduced Order dynamical system shows a divergence in time integration. A method of calibration based on the minimization of a linear functional of error, to the sensitivity of the modes, is proposed. The calibrated low order model is used to design a feedback control of cavity flows based on an observer design. For the experimental implementation of the controller, a state estimate based on the observed pressure measurements is obtained through a linear stochastic estimation. Finally the obtained control is introduced into the Direct Numerical Simulation to obtain a decrease in spectra of the cavity acoustic mode.

AMS subject classifications: 93C05, 93C10, 76N25

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1 Introduction

Self sustained instabilities in compressible cavities arise due to the impingement of the shear layer on the downstream edge of the cavity, and is caused by the energetic coherent

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structure, resulting in a pressure feedback mechanism which induces the far field noise in the cavity. The flow is characterized by a strong coupling between the hydrodynamic instability and the acoustic propagation which can be sometimes intense. Applications of such flows arise in airframe fuel vents, landing gears, weapon bays. Accurate modelling of the cavity noise is important from the point of view of a search for quieter aircrafts. The traditional tools for the modelling of the cavity like Direct Numerical Simulation (DNS) and Large Eddy Simulation (LES) pose difficulties in terms of computational resources, also the constraint increases when the ultimate aim of study is in the application of control. The self sustained instabilities in cavity has been studied in the past by many authors [1–5]. The principles of Reduced Order Modelling (ROM) based on Proper Orthogonal Decomposition (POD) and a Galerkin projection, has been in the past applied to model and control to flows with self-sustained instabilities, like wake behind cylinder [6] and high lift configurations [7]. Flow past an open cavity has already been studied using ROM by [8] and [9] but without any application to flow control. More recently, ROM for controlled configurations has been proposed by [10,11]. In [12] the ROM for flows issued from an experiment has been used to design a controller. The major hurdle in using the ROM for control applications is the accuracy of the model in predicting the dynamics of the system even for short periods, also difficulty arises when the control parameters are changed as in a real time simulation. Various numerical strategies termed as calibration techniques has been developed in the recent past based on the solution of an optimization problem and can be found in the works of [6,13–16]. A summary of the application of various method has been discussed in detail [17] where a method of calibration based on the Tikhonov regularization has been proposed. The main contribution of this paper is then to complete the full development as applied to cavities, like building up the ROM, including the effect of control, calibrating the model and finally performing control studies. We give further developments to the calibration technique by performing a sensitivity analysis to weight the importance of the errors in the different modes of the ROM, and accurately predict the dynamics of the system. The ROM thus obtained has been applied to perform the feedback control of cavity flows to obtain a reduction in the noise level of the cavity.

The paper is organized as follows. In Section 2 the derivation of the plant model used for control studies has been discussed. The general principles of model reduction is discussed in Subsection 2.1 where the usual ROM based on the principle of Galerkin projection is obtained as a special case. In Subsection 2.2 we discuss the general principles POD, followed by a discussion of the ROM for the isentropic equations in Subsection 2.2.1. Extension of ROM to include the effect of actuation is presented in Subsection 2.3. Section 3 concerns the calibration of the ROM to accurately represent the temporal dynamics, the various errors that define the dynamics of the ROM are presented in Subsection 3.1. A new approach to Tikhonov based regularization based on the sensitivity analysis is presented in Subsection 3.2 to obtain an accurate representation of the dynamics. The ROM has been applied to the cavity flow configuration in Section 4. Finally feedback control law based on the estimation of the observer dynamics has been presented in Section 5.