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A Positivity-Preserving Scheme for the Simulation of Streamer Discharges in Non-Attaching and Attaching Gases

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Abstract. Assumed having axial symmetry, the streamer discharge is often described by a fluid model in cylindrical coordinate system, which consists of convection dominated (diffusion) equations with source terms, coupled with a Poisson's equation. Without additional care for a stricter CFL condition or special treatment to the negative source term, popular methods used in streamer discharge simulations, e.g., FEM-FCT, FVM, cannot ensure the positivity of the particle densities for the cases in attaching gases. By introducing the positivity-preserving limiter proposed by Zhang and Shu [15] and Strang operator splitting, this paper proposes a finite difference scheme with a provable positivity-preserving property in cylindrical coordinate system, for the numerical simulation of streamer discharges in non-attaching and attaching gases. Numerical examples in non-attaching gas (N_2) and attaching gas (SF_6) are given to illustrate the effectiveness of the scheme.

AMS subject classifications: 65Z05, 65M06, 68U20

Key words: WENO finite difference, positivity-preserving, streamer discharge, numerical simulation.

1 Introduction

As the initial stage of various electrical discharges such as sparks and lightnings, streamer discharges happen in natural environment and many industrial applications everyday. Great efforts have been taken for the experimental study of streamer discharges over several decades [1]. However, due to the lack of rigorous measurement methods, the existing experiment data are still insufficient to build a clear picture of streamer discharges,

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which makes numerical simulations an important auxiliary tool to predict detailed physical quantities in the discharge channel. A better understanding on the physics of streamer formation and propagation may be achieved by comparing these numerical predictions with experimental observations.

The most frequently used model to describe streamer discharges is the fluid model, which consists of the particle density continuity equations (which are convection-dominated equations with source terms) coupled with a Poisson's equation with axial symmetries:

$$\frac{\partial n_e}{\partial t}+\frac{1}{r}\frac{\partial(rv_{er}n_e)}{\partial r}+\frac{\partial(v_{ez}n_e)}{\partial z}-\frac{D_r}{r}\frac{\partial}{\partial r}\left(r\frac{\partial n_e}{\partial r}\right)-D_z\frac{\partial^2 n_e}{\partial z^2}=(\alpha-\eta)n_e|\vec{v}_e|, \tag{1.1}$$

$$\frac{\partial n_p}{\partial t}+\frac{1}{r}\frac{\partial(rv_{pr}n_p)}{\partial r}+\frac{\partial(v_{pz}n_p)}{\partial z}=\alpha n_e|\vec{v}_e|, \tag{1.2}$$

$$\frac{\partial n_n}{\partial t}+\frac{1}{r}\frac{\partial(rv_{nr}n_p)}{\partial r}+\frac{\partial(v_{nz}n_p)}{\partial z}=\eta n_e|\vec{v}_e|, \tag{1.3}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\varepsilon_0\frac{\partial U}{\partial r}\right)+\frac{\partial}{\partial z}\left(\varepsilon_0\frac{\partial U}{\partial z}\right)=e_0(n_e+n_n-n_p), \tag{1.4}$$

$$\vec{E}=(E_r,E_z)^T=-\left(\frac{\partial U}{\partial r},\frac{\partial U}{\partial z}\right)^T, \quad |\vec{E}|=\sqrt{E_r^2+E_z^2}, \tag{1.5}$$

$$\vec{v}_{e,p,n}=\left(v_{(e,p,n)r},v_{(e,p,n)z}\right)^T=\mu_{e,p,n}(|\vec{E}|)\vec{E}, \quad |\vec{v}_e|=\sqrt{v_{er}^2+v_{ez}^2}, \tag{1.6}$$

where t denotes time, $r\in[0,a_1]$, $z\in[b_1,b_2]$, $a_1>0$, and $b_1,b_2\in\mathbb{R}$; $n_{e,p,n}$ are the densities of charged particles, $\mu_{e,p,n}$ are the movability coefficient; $\vec{v}_{e,p,n}$ is the drift velocity; D_r and D_z are the diffusion coefficients, the index e, p, n stand for electrons, positive ions, negative ions, respectively. U and $\vec{E}$ are the electrical potential and electric field, respectively; ε_0 is the dielectric coefficient in air; e_0 is the unit charge of an electron. α and η are measured by experiments and $\alpha>0$, $\eta>0$. They are functions of $|\vec{E}|/N$, i.e., electric field strength $|\vec{E}|$ divided by the neutral gas number density N, see Fig. 1 for an example; in addition, there exists such a critical value E_1 for each gas that

$$\begin{cases}\alpha\le\eta, & \text{if } |\vec{E}|\le \mathrm{E}_1;\\ \alpha>\eta, & \text{if } |\vec{E}|>\mathrm{E}_1.\end{cases} \tag{1.7}$$

By Eq (1.7), strictly speaking, the source term in Eq. (1.1) may be either negative or positive for both non-attaching and attaching gases. However, when the applied voltage is near or a little more than the breakdown voltage, for non-attaching gas, $\alpha-\eta$ is positive everywhere in the discharge domain; $\alpha-\eta\ll 0$ still exists for attaching gases, which leads to a negative source term in Eq. (1.1).

For several decades, researchers to the paradigm of streamer discharge simulations have been focusing on the solution of the convection dominated particle density continuity equations, especially on the discretization of the convection term. Due to the ionization and charge accumulation effect, the particle density profile at the streamer's head