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Numerical Buoyancy-Wave Model for Wave Stress and Drag Simulations in the Atmosphere

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Abstract. Orographic drag formation is investigated using a numerical wave model (NWM), based on the pressure-coordinate dynamics of non-hydrostatic HIRLAM. The surface drag, wave stress (vertical flux of horizontal momentum), and wave drag are split to the longitudinal and transverse components and presented as Fourier sums of their spectral amplitudes weighted with the power spectrum of relative orographic height. The NWM is accomplished, enabling a spectral investigation of the buoyancy wave stress, and drag generation by orography and is then applied to a cold front, characterised by low static stability of the upper troposphere, large vertical and directional wind variations, and intensive trapped wave generation downstream of obstacles. Resonances are discovered in the stress and drag spectra in the form of high narrow peaks. The stress conservation problem is revisited. Longitudinal stress conserves in unidirectional flow, 2D orography conditions, but becomes convergent for rotating wind or 3D orography. Even in the convergent case the vertical momentum flux from the troposphere to stratosphere remains substantial. The transverse stress never conserves. Disappearing at the surface and on the top, it realises the main momentum exchange between lower an upper parts of the troposphere. Existence of stationary stratospheric quasi-turbulence (SQT) is established above wind minimum in the stratosphere.

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1 Introduction

In recent years, wave and wave-originated drag modelling have become vital for applications of drag parameterisation in global circulation, numerical weather prediction, and climate studies [1–11]. Among different buoyancy waves, orographic waves are the most important drag source. The two kinds of drag that affect laminar atmospheric flow by terrain are surface drag and wave drag. Surface drag was first introduced by [12] and subsequently discussed in early studies by [13,14], and for three-dimensional orography, by [15]. It affects the near-surface momentum mainly (as it will be demonstrated hereafter, the surface drag will vanish above mountain crests, producing wind weakening in the front and the lees of mountains and yielding the blocking and envelope-orography formation). Wave drag is caused by the wave stress (which is the same as the wave-originated vertical flux of horizontal momentum) convergence in the upper-level turbulent layers, which are created by the same waves when they break. Such an upper-level drag mechanism was discussed in the pioneering work by [16] (see also [17, 18]), who showed that wave stress is conserved in mean zonal flow (now known as the Eliassen-Palm momentum flux conservation theorem), and proposed the wave breaking with following turbulent dissipation as a probable mechanism, capable of the upper-level momentum change explanation. As in this paper will be demonstrated, the major part of the stress will pass from the troposphere to the stratosphere without notable conversion even if it is not strictly conserved.

Whereas upper-level wave-breaking induced turbulence still remains beyond the capabilities of numerical weather prediction and climate models and requires sub-grid parameterisation, generation of surface drag and wave stress can be modelled numerically because contemporary non-hydrostatic models do provide the necessary resolution. The most straightforward method for stress and drag studies lies in modelling wave generation by orography. There exist a great number of research papers dedicated to wave generation by orography and wave-related drag problems. In some of the most cited works, the problem has been studied both numerically by integrating non-stationary, non-linear, and typically non-hydrostatic equations of atmospheric dynamics [19–29], and analytically by solving linear wave equations [30–38].

In the present paper, we will revisit the orographic drag generation problem using an approach for wave and wave-related drag study that combines the power of the linear spectral method with the flexibility of numerical approach. A previously developed numerical solution method [39], **RZ07** hereafter, for linear spectral wave equation is extended to include stress and drag computation in a general non-stationary case for any arbitrary thermal and wind stratification. The updated numerical wave model (**NWM**) is then applied to wave and accompanying stress and drag modelling in the real atmosphere. Using a friction-free, non-reflective (radiative) upper boundary condition, NWM can handle waves vertically up to the mesosphere. The departure wave equation in RZ07 is based on the Miller-Pearce [40,41] anelastic, non-hydrostatic, pressure-coordinate equations, previously implemented in non-hydrostatic HIRLAM [42, 43]. These equa-