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Simulation of Power-Law Fluid Flows in Two-Dimensional Square Cavity Using Multi-Relaxation-Time Lattice Boltzmann Method

Qiuxiang Li[1], Ning Hong[2], Baochang Shi[3]* and Zhenhua Chai[3]

[1] *State Key Laboratory of Coal Combustion, Huazhong University of Science and Technology, Wuhan, 430074, P.R. China.*
[2] *Department of Foundational Courses, Jiangcheng College, China University of Geosciences, Wuhan, 430200, P.R. China.*
[3] *School of Mathematics and Statistics, Huazhong University of Science and Technology, Wuhan, 430074, P.R. China.*

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Abstract. In this paper, the power-law fluid flows in a two-dimensional square cavity are investigated in detail with multi-relaxation-time lattice Boltzmann method (MRT-LBM). The influence of the Reynolds number (*Re*) and the power-law index (*n*) on the vortex strength, vortex position and velocity distribution are extensively studied. In our numerical simulations, *Re* is varied from 100 to 10000, and *n* is ranged from 0.25 to 1.75, covering both cases of shear-thinning and shear-thickening. Compared with the Newtonian fluid, numerical results show that the flow structure and number of vortex of power-law fluid are not only dependent on the Reynolds number, but also related to power-law index.

AMS subject classifications: 76A05, 76M28, 76D05, 81T80

Key words: Multi-relaxation-time lattice Boltzmann method, power-law fluid, shear-thinning, shear-thickening.

1 Introduction

The non-Newtonian fluids are widely observed in many fields of science and technology, such as food, petroleum, lubricants, geophysics, hydrogeology, chemistry, to name but a

*Corresponding author. *Email addresses:* sbchust@126.com (B. Shi), llqqxxx@126.com (Q. Li), hn_jxcy@126.com (N. Hong), hustczh@126.com (Z. Chai)

few. Moreover, many modern materials and manufacturing processes also require further understanding on the behavior of non-Newtonian fluids since their wide applications in practice [1,2]. Compared to the Newtonian fluids, the non-Newtonian fluids usually have a complicated constitutive equation, which may bring more difficulties in investigating non-Newtonian behavior with numerical methods. In the past years, some advanced or efficient methods have been developed to simulate the non-Newtonian fluid flows, such as finite element method [3], finite volume method [4], lattice Boltzmann method (LBM) [5] and smoothed particle hydrodynamics method (SPH) [6]. With the aid of the methods mentioned above, some complex flow features that differ from the Newtonian fluids have been reported.

In this paper, we will use a lattice Boltzmann method to study the behaviors of power-law non-Newtonian fluid flows in a two-dimensional square cavity. During the last two decades, the LBM, as a new mesoscopic method, has been proved to be a powerful numerical technique in simulating complex Newtonian and non-Newtonian fluid flows [7–9] and particularly successful in dealing with complex boundaries for its kinetic background [9–14]. Compared to some traditional numerical methods, another advantage of the LBM is that the stress tensor can be obtained locally from the non-equilibrium parts of the distribution functions [12,13]. Hence, the LBM is considered to offer excellent possibilities for simulating non-Newtonian flows [13].

A popular lattice Boltzmann model is the so-called lattice Bhatnagar-Gross-Kook model (LBGK) [14, 15], which has been widely applied to study complex flows. However, LBGK model is usually unstable when the relaxation time is close to 0.5. One way to overcome this shortcoming of the LBGK model is to use a multiple-relaxation-time (MRT) model, or generalized lattice Boltzmann model, which was originally proposed by d'Humières [16] and further developed by Lallemand and Luo [17]. Compared with the LBGK model, MRT model can improve the numerical stability and reduce the unphysical oscillations for some flows.

The lid-driven flow in a two-dimensional (2D) square cavity, as a classic benchmark problem in fluid mechanics, has been widely studied by many researchers [18–26] in the past decades. Driven cavity flow is of great importance because it can offer an ideal framework in which meaningful and detailed comparisons can be made between results obtained from theory and computation [20]. Besides, this problem has a simple geometry, but covers a wide range of complex hydrodynamics encompassing eddies, secondary flows, instability and transition, which are of great importance to the basic study of fluid mechanics. However, to the best of the knowledge of the authors, most available works are limited to the Newtonian fluids and only few works associated with the non-Newtonian (power-law) fluid flows in a square cavity have been reported. To fill the gap, in the present work, the LBM coupling with MRT model is used to simulate power-law non-Newtonian fluid flows in a two-dimensional square cavity (see Fig. 1). We intend to explore the complex phenomena of non-Newtonian fluid flows and investigate non-Newtonian effects in the lid-driven cavity flows, a detailed comparison of the non-Newtonian results with Newtonian results have been done.