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# An Entropic Scheme for an Angular Moment Model for the Classical Fokker-Planck-Landau Equation of Electrons

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**Abstract.** In plasma physics domain, the electron transport is described with the Fokker-Planck-Landau equation. The direct numerical solution of the kinetic equation is usually intractable due to the large number of independent variables. That is why we propose in this paper a new model whose derivation is based on an angular closure in the phase space and retains only the energy of particles as kinetic dimension. To find a solution compatible with physics conditions, the closure of the moment system is obtained under a minimum entropy principle. This model is proved to satisfy the fundamental properties like a H theorem. Moreover an entropic discretization in the velocity variable is proposed on the semi-discrete model. Finally, we validate on numerical test cases the fundamental properties of the full discrete model.

**AMS subject classifications**: 35Q84, 65L04, 76Y05

**Key words**: Entropy minimization, Landau-Fokker-Planck equation, moment systems, entropic scheme.

## 1 Introduction

Classically in kinetic theory, a Fokker-Planck equation is used to describe the evolution of different species in a collisional plasma [8, 14]. These charged particles can interact through long-range Coulomb interactions. More precisely the solutions of the kinetic equations are non-negative distribution functions $f_\alpha(t,x,v)$ specifying the density of each specie $\alpha$ of particles with velocity $v$ at time $t$ and position $x$. For the sake of simplicity, we assume in this paper that the plasma consists of electrons and one ion species which

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are considered fix in the plasma. In order to approximate the solution of such problems, many computational methods have been developed up to now.

There is a variety of models that are used to describe electrons transport: the hydrodynamic ones [9–11] and the kinetic ones [4, 5, 12, 20, 21, 32]. But in the present paper, we consider an intermediate description between fluid and kinetic level. Let us consider $f^0$ the zeroth order moment of the distribution function $f$ (isotropic part), its first order moment $f^1$ and its second order moment $f^2$ with respect to the angular part of the velocity variable. The usual model employed in plasma physic can be expressed such as

$$\begin{cases} \partial_t f^0 + \zeta \partial_x f^1 = Q^0(f^0), \\ \partial_t f^1 + \zeta \partial_x f^2 = -\kappa f^1, \end{cases} \tag{1.1}$$

where $\kappa \in \mathbb{R}^+$ and $\zeta$ is the microscopic energy variable. Here $Q^0(f^0)$ represents an approximation for the electron-electron collision operator. In plasma physics, classical approximation lead to consider that the main contribution for the electron-electron collision operator comes from the isotropic part, that is why here $Q^0$ depends only of $f^0$. This model allows mainly to conserve fundamental properties such as the mass and energy conservation and the entropy dissipation. However this model is ill posed. Indeed whatever the closure chosen, this model does not preserve the realizability domain. Of course, except this model, some kinetic models like Fokker Planck model exist but computations are too much expensive and mainly kinetic codes can not run many collisional times. Most of them are restricted to some decade of collisional time in practice. For example there exists a kinetic code called *KETS* used in [20, 21] and cross-validated with an implicit Vlasov-Fokker-Planck code called *IMPACT* developed by Kingham and Bell [27]. Beyond a few decade of collisional time, *KETS* code becomes too costly. To reduce the computational time, the plasma can be described by fluid models. For example in [9–11] a bi-fluid compressible Euler model coupled with the Poisson equation is considered. However, for the new high energy target drivers, the kinetic effects are too important to neglect them and replace kinetic calculus by usual macroscopic Euler Models with $T_i$-$T_e$ temperatures.

That is why to preserve the realizability domain, we consider in this paper a new approximation of the electron-electron collision operator to correct the model (1.1). This new continuous model satisfies fundamental properties (i.e. conservation laws, entropy dissipation and conservation of the realizability domain). The microscopic velocity is written in spherical coordinates and the model is written by considering moments system with respect to the angular variable. But the choice of the closure on the angle variable is crucial to guarantee reasonable properties for the resulting model. For example, the $P_1$ model [23] neither satisfies positivity of the underlying distribution function of electrons and nor entropy dissipation. Indeed, the closure chosen defines the distribution function by a polynomial function depending on the angle variable. However in [25], a modification of $P_N$ model is proposed to correct this defect. Nevertheless the positive $P_N$ model does not dissipate the Boltzmann entropy. For this reason there is a considerable motivation to develop an other moment closure. That is why we will derive a