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# Chaotic Lid-Driven Square Cavity Flows at Extreme Reynolds Numbers

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**Abstract.** This paper investigates the chaotic lid-driven square cavity flows at extreme Reynolds numbers. Several observations have been made from this study. Firstly, at extreme Reynolds numbers two principles add at the genesis of tiny, loose counterclockwise- or clockwise-rotating eddies. One concerns the arousing of them owing to the influence of the clockwise- or counterclockwise currents nearby; the other, the arousing of counterclockwise-rotating eddies near attached to the moving (lid) top wall which moves from left to right. Secondly, unexpectedly, the kinetic energy soon reaches the qualitative temporal limit's pace, fluctuating briskly, randomly inside the total kinetic energy range, fluctuations which concentrate on two distinct fragments: one on its upper side, the upper fragment, the other on its lower side, the lower fragment, switching briskly, randomly from each other; and further on many small fragments arousing randomly within both, switching briskly, randomly from one another. As the Reynolds number $\mathrm{Re}\to\infty$, both distance and then close, and the kinetic energy fluctuates shorter and shorter at the upper fragment and longer and longer at the lower fragment, displaying tall high spikes which enlarge and then disappear. As the time $t\to\infty$ (at the Reynolds number Re fixed) they recur from time to time with roughly the same amplitude. For the most part, at the upper fragment the leading eddy rotates clockwise, and at the lower fragment, in stark contrast, it rotates counterclockwise. At $\mathrm{Re}=10^9$ the leading eddy — at its qualitative temporal limit's pace — appears to rotate solely counterclockwise.

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**Key words**: Navier-Stokes equations, lid-driven square cavity flows, chaos.

## 1 Introduction

As the Reynolds number $\mathrm{Re}\to\infty$ the temporal limit [13, p. 659] (at the Reynolds number Re fixed and as the time $t\to\infty$) of the lid-driven square cavity flow evolves from station-

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ary to periodic and then to aperiodic, which means simply that it has definitely lost but still resembles somewhat periodicity. Yet afterwards, it definitely looses aperiodicity and becomes chaotic.

Specifically, for $\mathrm{Re} \leq 7{,}307.75$, it is stationary; for $7{,}308 \leq \mathrm{Re} \leq 13{,}393.5$, periodic; for $13{,}393.75 \leq \mathrm{Re} \leq 200{,}000$, aperiodic. Yet afterwards, at $\mathrm{Re} = 500{,}000$, it is chaotic. So it switches from stationary to periodic somewhere between $\mathrm{Re} = 7{,}307.75$ and $\mathrm{Re} = 7{,}308$; from periodic to aperiodic, somewhere between $\mathrm{Re} = 13{,}393.5$ and $\mathrm{Re} = 13{,}393.75$ [26]; from aperiodic to chaotic, somewhere between $\mathrm{Re} = 200{,}000$ and $\mathrm{Re} = 500{,}000$ [27].

Throughout, the moving (lid) top wall moves from left to right. And until $\mathrm{Re}=200{,}000$ the leading eddy rotates solely clockwise. But at $\mathrm{Re} = 500{,}000$ it rotates sometimes clockwise and at other times counterclockwise, mostly equally, a competition for becoming the leading eddy taking place before switching from each other [27].

A combination of known methods [25] is used to discretize and solve the Navier-Stokes equations: the linear Lin$\theta^*$-scheme [44] (a variant of the nonlinear $\theta$-scheme [30], [23, p. 460]), an orthogonal projection algorithm [38], the Conjugate Gradient method [12], the Bi-CGSTAB method [50], the Fast Fourier Transform method [18,47,48], SuperLU [14] — and the incremental unknowns method [4–11, 19–28, 31, 37, 45, 46, 49] as a spatial preconditioner. The linear Lin$\theta^*$-scheme is used for the temporal discretization — $\Delta t$ is the time step — and a staggered marker-and-cell (MAC) mesh with finite-differences [32] is used for the spatial discretization — $h = 1/256 = 0.00390625$ is the spatial mesh size. At each temporal iteration two generalized Stokes equations and two linear elliptic equations with variable coefficients must be solved.

Yet, how the temporal limit behaves for extreme Reynolds numbers — has it been definitely reached — and what is the time needed to reach it?

To gain insight on these questions, let us consider the extreme cases $\mathrm{Re} = 10^6$, $10^8$, $5 \cdot 10^8$, $10^9$. As before [26, 27], a Direct Numerical Simulation (DNS) which runs from $t = 0$ to a sufficiently long time $t_\infty$ simulates the flow, and then direct observations of it for the most part past the time $t_\infty$ are at the core of the temporal limit's study. At $\mathrm{Re} = 10^6$ as at $\mathrm{Re}=500{,}000$ the time step $\Delta t=2h=0.0078125$ and $t_\infty=150{,}000$, a DNS involving a total of 19,200,000 temporal iterations. But at $\mathrm{Re}=10^8$, $5 \cdot 10^8$, $10^9$ the time step must be reduced to $\Delta t = h = 0.00390625$, and then because of computational costs, $t_\infty$ is reduced sometimes to $t_\infty = 50{,}000$, a DNS involving a total of 12,800,000 temporal iterations. At each Reynolds number two DNS are conducted: (1) fluid starting from data and (2) fluid starting from rest. For fluid starting from data the initial condition is the numerical solution computed at some time at a prior Reynolds number.

At extreme Reynolds numbers the flow is difficult to study mainly for two reasons. First, for the leading eddy to switch from rotating counterclockwise to clockwise, the DNS ought to run for a sufficiently long time interval, the switching occurring somewhere in between. But to determine what triggers the switching, it must be restarted shortly before the switching and therefore run for a short time interval, highly likely missing the switching. Second, all the restartings coincide at the beginning for a short while, then progressively differ for a short while — and afterwards significantly differ,