

## A Penalty Optimization Algorithm for Eigenmode Optimization Problem Using Sensitivity Analysis

Zhengfang Zhang<sup>1,\*</sup>, Weifeng Chen<sup>2</sup> and Xiaoliang Cheng<sup>3</sup>

<sup>1</sup> College of Science, Hangzhou Dianzi University, Hangzhou 310018, P.R. China.

<sup>2</sup> School of Information, Zhejiang University of Finance and Economics, Hangzhou 310018, P.R. China.

<sup>3</sup> Department of Mathematics, Zhejiang University, Hangzhou 310027, P.R. China.

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**Abstract.** This paper investigates the eigenmode optimization problem governed by the scalar Helmholtz equation in continuum system in which the computed eigenmode approaches the prescribed eigenmode in the whole domain. The first variation for the eigenmode optimization problem is evaluated by the quadratic penalty method, the adjoint variable method, and the formula based on sensitivity analysis. A penalty optimization algorithm is proposed, in which the density evolution is accomplished by introducing an artificial time term and solving an additional ordinary differential equation. The validity of the presented algorithm is confirmed by numerical results of the first and second eigenmode optimizations in 1D and 2D problems.

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## 1 Introduction

To investigate the structural dynamics characteristics of mechanical systems, the use of modal analysis is widely applied. The modal pairs consist of eigenfrequencies and eigenmodes are used to identify the cause of vibrational problems. By making use of the eigenpairs, one can evaluate the change of dynamical properties when mass and/or stiffness is added or subtracted without changing the structure.

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\*Corresponding author. *Email addresses:* zhengfangzhang@hdu.edu.cn (Z. Zhang), chenweifeng@cad.zju.edu.cn (W. Chen), xiaoliangcheng@zju.edu.cn (X. Cheng)

The vibration control at low frequency is an important research field [25, 26]. To design devices with the specified dynamic structural optimization, various numerical optimization algorithms are established with respect to the structure design variables such as size, shape and topology [1, 8, 19, 30]. For avoidance of resonance, one effective method is to maximize the lowest eigenfrequency to determine the shape of the vibrating membrane composed of materials with different densities [6, 15, 41]. Osher and Santosa [28] solve the model problem by the level set method [29], combining the variational level set method [42] and the projection gradient method [31]. A monotonic algorithm [40], based on resorting order, can efficiently deal with the eigenvalue optimizations of multi-density materials. Another kind of eigenvalue optimization problems arises from a large class of problems in the field of boundary control or reinforcement [5, 33]. Cox and Uhlig [7] analyze the existence and convergence of the eigenvalue boundary optimization, establish the pointwise optimal condition, and test a pair of numerical methods. Zhang and Cheng [39] propose a boundary piecewise constant level set method to parameterize the boundary condition and convert the eigenvalue optimization problem to rely on a parameter instead of the boundary geometry, which generalizes the classical piecewise constant level set method [34].

Eigenmode optimization, another branch of vibration optimization problems, is of great importance. Typical examples are mechanical resonators that are used as sensors, oscillators, filters and actuators. In this type of resonator, the eigenmode that dominates the shape of deformation against the external periodic load is an important design factor in addition to the resonance frequency. One popular technique is the topology optimization [8, 19, 21, 23], based on homogenization theory proposed by Bendsøe and Kikuchi [2]. In this technique, the material distribution is formulated with parameters in periodic microstructures. Another currently used method is the SIMP (solid isotropic material with penalization) method [9, 16, 18, 36]. The basic idea of SIMP is taking use of a fictitious isotropic material whose elasticity tensor is expressed by an exponent parameter and assumed to be a function of penalized material density. However, these topology optimization methods cause numerical problems such as checkerboard patterns, grey scales and artificial parameter dependence.

Sensitivity analysis is a fundamental tool for the optimization problems in both the discrete form and the continuum form. For the eigenmode optimization problem, some researchers use eigenmode sensitivity analysis of matrices, which are derived by the finite element method (FEM) discretization of original continuum problem [10, 22, 38]. In [21], vibrating structures with specified eigenfrequencies and eigenmode shapes are investigated. Maeda et al. propose a new topology optimization method based on homogenization theory and derive the sensitivity of the eigenvalue and eigenmode with respect to design variable after FEM discretization is performed. In continuum form, to the best of the authors' knowledge, sensitivity analysis for eigenmode optimization problem are reported by [17, 35]. In [17], Inzarulfaisham and Azegami evaluate the shape gradient for the boundary shape optimization problem with optimality conditions obtained by the adjoint variable method, the Lagrange multiplier method and the formula for the mate-