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Remapping-Free Adaptive GRP Method for Multi-Fluid Flows I: One Dimensional Euler Equations

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Abstract. In this paper, a remapping-free adaptive GRP method for one dimensional (1-D) compressible flows is developed. Based on the framework of finite volume method, the 1-D Euler equations are discretized on moving volumes and the resulting numerical fluxes are computed directly by the GRP method. Thus the remapping process in the earlier adaptive GRP algorithm [17,18] is omitted. By adopting a flexible moving mesh strategy, this method could be applied for multi-fluid problems. The interface of two fluids will be kept at the node of computational grids and the GRP solver is extended at the material interfaces of multi-fluid flows accordingly. Some typical numerical tests show competitive performances of the new method, especially for contact discontinuities of one fluid cases and the material interface tracking of multi-fluid cases.

AMS subject classifications: 76M12, 76N15, 76T99

Key words: The GRP method, multi-fluid flows, the Euler equations, the adaptive mesh method.

1 Introduction

Compressible multi-fluid flows can be found in a variety of scientific and engineering problems, and they are characterized by the interaction of shock waves and material interfaces. On account of those complicated fluid phenomena including strong shocks, contact discontinuities, instabilities of material interfaces, mixing processes and so on, developing numerically accurate and computationally efficient algorithms is still one of

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the challenging issues for multi-fluid simulations [1–3, 14, 20, 27, 28, 33, 34]. In this paper, we focus on the numerical methods for multi-fluid flows consisting of pure fluids separated by material interfaces.

The computation of numerical fluxes is always an important issue of numerical methods for compressible fluid dynamics. The generalized Riemann problem (GRP) method which was developed as an analytic second order accurate extension of the Godunov scheme is one of successful numerical methods to solve this problem. The basic idea of the GRP scheme consists of replacing the exact solution by a piecewise linear function and analytically solving a generalized Riemann problem at each cell interface to yield numerical fluxes, to achieve second order accuracy both in space and time. The GRP scheme was originally developed for compressible flows based on the Lagrangian formulation and the Eulerian version was always derived from the Lagrangian case [4, 7]. Then a direct Eulerian GRP scheme was presented in [8, 9, 23–25], aiming at getting rid of the auxiliary Lagrangian scheme and solving the 1-D generalized Riemann problem directly in the Eulerian frame by employing the regularity property of the Riemann invariants. Theoretically, a close coupling between the spatial and temporal evolution is recovered through the analysis of detailed wave interactions in the GRP scheme. The schemes based on the GRP method have been applied successfully to many engineering problems [5, 6, 15, 16, 23].

Computational mesh is another vital issue of numerical algorithms. In the physical and engineering problems, dynamically singular or nearly singular solutions, such as shock waves, boundary layers, etc., take place in fairly local regions. The numerical investigation of such problems may require extremely fine grids over such local domains to resolve large solution variations. Comparing with uniform grids, partly dense grids will improve the resolution of local regions and decrease the computational costs if the grids are moved at a selected adaptive speed at each time step. The adaptive mesh method [30] is one of effective moving mesh methods. A lot of important theoretical and computational progresses for partial differential equations demonstrate the advantages of the time-dependent adaptive mesh methods [10–13, 19, 21, 22, 31, 32, 36].

Based on the idea of adaptive mesh methods, the one-dimensional and two-dimensional adaptive GRP schemes by combining the Eulerian GRP scheme with the adaptive moving mesh method are developed in [17, 18]. Besides the PDE evolution, mesh redistribution is introduced in the adaptive GRP method in order to provide enough grids for specific structure of solutions such as shock waves. Thus the adaptive GRP method could improve the resolution for numerical solutions and reduce possible oscillations effectively. The computational mesh at different time steps can be generated adaptively based on a certain moving mesh method. Then physical variables and their slopes on new grids need to be updated by conservative interpolations.

On account of the complicated procedure for updating variables, this paper will develop a remapping-free adaptive GRP method. Based on the framework of finite volume method, the 1-D Euler equations are discretized on moving meshes and the resulting fluxes are computed directly by the GRP method. The material interfaces are moving