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A Conservative Local Discontinuous Galerkin Method for the Schrödinger-KdV System

Yinhua Xia[1,*] and Yan Xu[1]

[1] *School of Mathematical Sciences, University of Science and Technology of China, Hefei, Anhui 230026, P.R. China.*

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Abstract. In this paper we develop a conservative local discontinuous Galerkin (LDG) method for the Schrödinger-Korteweg-de Vries (Sch-KdV) system, which arises in various physical contexts as a model for the interaction of long and short nonlinear waves. Conservative quantities in the discrete version of the number of plasmons, energy of the oscillations and the number of particles are proved for the LDG scheme of the Sch-KdV system. Semi-implicit time discretization is adopted to relax the time step constraint from the high order spatial derivatives. Numerical results for accuracy tests of stationary traveling soliton, and the collision of solitons are shown. Numerical experiments illustrate the accuracy and capability of the method.

AMS subject classifications: 65M60, 35Q53, 35Q55

Key words: Schrödinger-KdV system, the conservative local discontinuous Galerkin method, semi-implicit time discretization, conservative quantities.

1 Introduction

We consider the long wave and short wave interaction system

$$\mathrm{i}(u_t+c_1u_x)+\delta_1u_{xx}=\alpha uv, \tag{1.1a}$$

$$v_t+c_2v_x+\delta_2v_{xxx}+\gamma(v^2)_x=\beta(|u|^2)_x, \tag{1.1b}$$

where u is a complex-valued function of real variables x and t, v is a real-valued function of x and t, and constants c_i, δ_i, α, β, γ are real. The Sch-KdV system arises in various physics contexts as a model for interaction of long and short nonlinear waves. For example, Kawahara et al. [13] derived the system as a model for the interaction between long gravity waves and capillary waves on the surface of shallow water. In [2, 14, 15],

*Corresponding author. *Email addresses:* yhxia@ustc.edu.cn (Y. Xia), yxu@ustc.edu.cn (Y. Xu)

the system is derived for resonant ion-sound/Langmuir-wave interactions in plasmas. Similarly, one can obtain the system as the unidirectional reduction of a model for the resonant interaction of acoustic and optical modes in a diatomic lattice [21]. In this system, the short wave is usually described by the Schrödinger type equation and the long wave is described by the wave equation accompanied with a dispersive term. When the resonance condition $c_1 = c_2$ holds, this system is known as the coupled Schrödinger-Korteweg-de Vries (Sch-KdV) system. Finite element methods and meshless methods have been considered in [3,4,12].

If $\delta_2 \neq 0$, then by using the suitable transformation in [1,2], the Sch-KdV system (1.1) is reduced to

$$\mathrm{i}u_t + \frac{3}{2}u_{xx} = \frac{1}{2}uv, \tag{1.2a}$$

$$v_t + \frac{1}{2}v_{xxx} + \frac{1}{2}(v^2)_x = -\frac{1}{2}(|u|^2)_x. \tag{1.2b}$$

We will also have occasion to consider the case when $\delta_2 = \gamma = 0$ in (1.1). In this case, the system (1.1) is reduced to the form

$$\mathrm{i}u_t + u_{xx} = uv, \tag{1.3a}$$

$$v_t = -(|u|^2)_x. \tag{1.3b}$$

This system has independent mathematical interest because it has been shown to be a completely integrable structure [5].

In this paper, we will present the conservative local discontinuous Galerkin (LDG) scheme for the system (1.2) and (1.3). For the general system (1.1), the LDG scheme is similar to these two special cases. Conservative quantities of the number of plasmons, energy of the oscillations and the number of particles in discrete versions are preserved in the LDG scheme for the Sch-KdV system. The conservative scheme for Sch-KdV system is very crucial for the long time simulation, especially for the integrable system (1.3).

The discontinuous Galerkin (DG) method is a class of finite element methods, using discontinuous, piecewise polynomials as the solution and the test spaces. It was first designed as a method for solving hyperbolic conservative quantities containing only the first order spatial derivatives, e.g. Reed and Hill [16] for solving linear equations, and Cockburn et al. [7–10] for solving nonlinear equations. The LDG method is an extension of the discontinuous Galerkin method aimed at solving partial differential equations (PDEs) containing higher than first order spatial derivatives. The idea of the LDG method is to rewrite the equations with higher order derivatives into a first order system, then apply the DG method on the system. The design of the numerical fluxes is the key ingredient to ensure stability. The LDG methods for a general KdV type equation with third order derivatives [22] and general Schrödinger equation [19] have been designed, in which the dissipation mechanism were introduced in the scheme. Recently, a conservative DG method is proposed in [6], in which the conservation properties are numerically