

A Runge Kutta Discontinuous Galerkin Method for Lagrangian Compressible Euler Equations in Two-Dimensions

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Abstract. This paper presents a new Lagrangian type scheme for solving the Euler equations of compressible gas dynamics. In this new scheme the system of equations is discretized by Runge-Kutta Discontinuous Galerkin (RKDG) method, and the mesh moves with the fluid flow. The scheme is conservative for the mass, momentum and total energy and maintains second-order accuracy. The scheme avoids solving the geometrical part and has free parameters. Results of some numerical tests are presented to demonstrate the accuracy and the non-oscillatory property of the scheme.

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1 Introduction

In determining a numerical method for the multi-dimensional fluid flow, there are two typical choices. One is the Lagrangian framework in which the mesh is embedded in the

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fluid and moves with it. And the other is the Eulerian framework in which the mesh is treated as a fixed reference frame when the fluid moves. More generally, the grid points may be moved in some arbitrarily specified way that it is called the Arbitrary Lagrangian-Eulerian method (ALE). Most ALE algorithms consist of three steps. One is the Lagrangian step in which the solution and the grid are updated. The second step is a rezoning mesh in which the grid nodes are moved to a more optimal position and the third is remapping values in which the Lagrangian solution values are transferred to the new grid. In the numerical simulations of multi-material compressible fluid flows, the difficulty is how to handle the moving medium interface. Since the Lagrangian method can calculate fluid interface clearly, we present a new Lagrangian type scheme for solving the Euler equations of compressible gas dynamics in this paper.

An important point of constructing a Lagrangian discrete scheme is to decide where to locate degrees of freedom. It is generally separated into two kinds. One is the staggered scheme in which the velocity is defined at the nodes while the other variables are located inside the cells. This type of scheme was first introduced by von Neumann and Richtmyer in [2] for one-dimensional flows. The two-dimensional scheme was extended by Wilkins in [23] based on an internal energy formulation. The scheme was not conservative and admitted numerical spurious modes. In spite of these drawbacks, the scheme has been widely used for many years. Of course, many improvements have been made in order to solve the previous problems. Caramana and Shashkov constructed a staggered scheme which ensures the conservation of total energy in [1]. In [3], based on a mimetic finite difference scheme, Campbell and Shashkov improved the discretization of artificial viscosity so that the staggered Lagrangian scheme is an accurate and robust method. The other is the cell-centered scheme in which all variables are defined inside the cells. Here we concentrate our interests in the cell-centered schemes. This is because the staggered Lagrangian schemes use different control volumes for primary variables, it is very difficult to construct coherent high-order schemes for all these variables. On the contrary, the cell-centered Lagrangian schemes use only one control volume for primary variables, thus it is possible to construct coherent high-order schemes for all these variables.

In general, there are two kinds of the cell-centered Lagrangian schemes.

The first is that the mesh moves with the flow velocity (e.g. [4-8]). In [4], Cheng and Shu developed a class of Lagrangian cell-centered schemes on quadrilateral meshes. Their schemes are based on the finite volume method and achieve a higher order accuracy by using the high-order ENO reconstruction. The schemes are conservative for the density, momentum and total energy, are essentially non-oscillatory, have free parameters, and can maintain formal high order accuracy both in space and time. But the ENO reconstruction requires the information from the surrounding cells. Therefore, this method has less compactness for high order schemes. In [6], Maire et al. developed a new Lagrangian cell-centered scheme. In their scheme the vertex velocities and numerical fluxes through the cell interfaces are not computed independently as usual but in a consistent manner. The scheme feature is the introduction of four pressures on each