Numerical Approximation of a Compressible Multiphase System

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Abstract. The numerical simulation of non conservative system is a difficult challenge for two reasons at least. The first one is that it is not possible to derive jump relations directly from conservation principles, so that in general, if the model description is non ambiguous for smooth solutions, this is no longer the case for discontinuous solutions. From the numerical view point, this leads to the following situation: if a scheme is stable, its limit for mesh convergence will depend on its dissipative structure. This is well known since at least [1]. In this paper we are interested in the "dual" problem: given a system in non conservative form *and* consistent jump relations, how can we construct a numerical scheme that will, for mesh convergence, provide limit solutions that are the exact solution of the problem. In order to investigate this problem, we consider a multiphase flow model for which jump relations are known. Our scheme is an hybridation of Glimm scheme and Roe scheme.

Nomenclature

- α_i: volume fraction of phase i;

- ρ_i: density of phase i; $\rho = \sum_i \alpha_i \rho_i$: average density,

- $\tau_i = 1/\rho_i$: specific volume of phase i; $\tau = 1/\rho$: specific volume,

- $Y_i = \frac{\alpha_i \rho_i}{\rho}$: mass fraction of phase i;

- u: average velocity;

- p: pressure, p_i pressure of phase i;

- s specific entropy, s_i specific entropy of phase i, $s = \sum_i Y_i s_i$;

- ε_i: specific internal energy of phase i;

- e_i: internal energy of phase i, $e_i = \rho_i \varepsilon_i$;

- T_i: temperature of phase i;

- $e = \sum_i \alpha_i e_i$: internal energy; $E = e + \frac{1}{2}\rho u^2$: total energy

- $\kappa_i = \dfrac{\partial p_i}{\partial e_i}$, $\chi_i = \dfrac{\partial p_i}{\partial \rho_i}$;

- a: speed of sound, a_i speed of sound of phase i.

1 Introduction

In many applications, one needs to consider compressible flows where the fluid is made of several non mixable phases. Examples can been found in the nuclear industry, the oil industry, for engines, etc. Another class of applications can be found in the case of high explosives. In that case, the media is made of several non mixable materials that are so intimately mixed that their exchange surface is very large. Such a fluid can be modeled by two compressible fluids, each having its own equation of state, thus its own pressure and possibly its own velocity. However, in the case of a large inter-facial area, it is legitimate to assume that the phase pressures and velocities are identical. The same situation occur for atomized flows.

The model in this case cannot be the simple model of two mass conservation equations (one for each phase), the momentum conservation equation, a total energy equation and a last one describing the evolution of the fluid composition written as a simple transport equation. In fact, in the physical model, one may encounter smooth variations of the volume fractions. In that case, when a shock wave is moving, this implies that the fluids can be compressed according to their acoustic impedance. A model that describes such a situation is the Kapila model [2] which can be derived from variants of the Baer and Nunziato [3] model by means of asymptotic expansions, see [4]. Here the small parameter is related to the inverse of the inter-facial area. The system of PDEs of the Kapila model is given in section 2. It is written in non conservation form, hence it cannot describe the structure of shock waves: the classical Rankine-Hugoniot relations do not hold, and the derivation of jump relation cannot be obtained using the standard techniques.

However, in [5], R. Saurel and coauthors have derived from some heuristic arguments a series of jump relations. Basically, for n phase flows, one has for each phase the classical Hugoniot relations, supplemented by the fact one has a single pressure. These relations satisfies all the requirements, in particular for weak shocks, the Hugoniot curves are tangent to the isentropes. Last, these relations have been validated against numerous experimental test cases with very severe conditions.