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An Unpreconditioned Boundary-Integral for Iterative Solution of Scattering Problems with Non-Constant Leontovitch Impedance Boundary Conditions

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Abstract. This paper concerns the electromagnetic scattering by arbitrary shaped three dimensional imperfectly conducting objects modeled with non-constant Leontovitch impedance boundary condition. It has two objectives. Firstly, the intrinsically well-conditioned integral equation (noted GCSIE) proposed in [30] is described focusing on its discretization. Secondly, we highlight the potential of this method by comparison with two other methods, the first being a two currents formulation in which the impedance condition is implicitly imposed and whose the convergence is quasi-optimal for Lipschitz polyhedron, the second being a CFIE-like formulation [14]. In particular, we prove that the new approach is less costly in term of CPU time and gives a more accurate solution than that obtained from the CFIE formulation. Finally, as expected, It is demonstrated that no preconditioner is needed for this formulation.

AMS subject classifications: 65R20, 15A12, 65N38, 65F10, 65Z05

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1 Introduction

The boundary integral methods (BIM) are commonly used for solving scattering problems of arbitrarily shaped three-dimensional obstacles and also for antenna design. Their

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popularity is due to a combination of many factors. Firstly, the solutions of BIM fulfill causality and radiation conditions automatically. Secondly, it is only necessary to discretize the boundaries of the computational domain and the simulation requires a smaller number of unknowns than finite element methods or finite difference methods. One of the main drawbacks of using a BIM is that after discretization it results in a dense system of linear equations. If a large number of unknowns is involved, the only possibility is to use iterative solvers coupled with a fast matrix-vector multiplication. However, with such a tool various situations can be analyzed, for instance conductor obstacles, dielectric homogeneous obstacles and also imperfectly conductor materials. We focus our attention on imperfectly conducting materials. This type of materials is generally taken into account by imposing an impedance boundary condition like the Leontovitch condition [22] on the surface of the object. Such situations occur in radar applications: objects are often partially coated by a thin dielectric layer to reduce the radar cross section of scattering waves. Another domain of application of such boundary conditions is their use as an absorbing boundary condition to limit the computational domain. Several boundary integral formulations can be derived to solve such impedance problems. Most representatives are formulations using one current as the unknowns proposed in [1,5,14,26,29], the system of integral equations based on the minimization of a quadratic functional in [13] and a formulation keeping the two currents as unknowns obtained, in particular, in [21]. The equivalent currents are discretized by a boundary-element method over a triangular mesh on the surface. Finally, we obtain a dense linear system. When the characteristic size of the obstacle is about six times the wavelength, the solutions can be computed by direct methods with high performance parallel codes. However, if the size of the obstacle increases, the solutions can only be obtained by means of some iterative methods coupled with the multilevel fast multipole method (noted MFMM). The convergence of the iterative methods is directly linked to the choice of the integral formulation. So, the main difficulty is to choose the best boundary equation in the sense that this equation gives rise to a well-conditioned linear system and also in the sense that the solutions must be accurate. An algebraic preconditioner [11] is generally used to improve the convergence of the iterative solver. Unfortunately, this kind of approach loses its effectiveness when the frequency increases or the meshes become denser with respect to the wavelength.

Recently, another alternative is emerged. It consists of constructing new integral equations that give rise to intrinsically well-conditioned linear systems. The genesis of these techniques are the work of D. Levadoux at ONERA. Indeed, in his thesis [23–25], he initiated a new integral formalism known as GCSIE (Generalized Combined Source Integral Equation) in which he has combined pseudo-differential mathematical analysis and physical characteristics of waves to obtain integral equations well-adapted to an iterative solution. These works have led to the emergence of a general formalism of construction which has been used, with success, for many problems in acoustics and in electromagnetism. The new formalism depends on the choice of an operator. This one aims to be a good approximation of the admittance in the perfectly metallic case. Several propositions have already made to achieve this goal [3,4,6,7,18,31]. The numerical results are