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Multi-Phase Texture Segmentation Using Gabor Features Histograms Based on Wasserstein Distance

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Abstract. We present a multi-phase image segmentation method based on the histogram of the Gabor feature space, which consists of a set of Gabor-filter responses with various orientations, scales and frequencies. Our model replaces the error function term in the original fuzzy region competition model with squared 2-Wasserstein distance function, which is a metric to measure the distance of two histograms. The energy functional is minimized by alternative minimization method and the existence of closed-form solutions is guaranteed when the exponent of the fuzzy membership term being 1 or 2. We test our model on both simple synthetic texture images and complex natural images with two or more phases. Experimental results are shown and compared to other recent results.

AMS subject classifications: 68U10

Key words: Multi-phase texture segmentation, Wasserstein distance, Gabor filter, Mumford-Shah model.

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1 Introduction

Image Segmentation has been one of the most talked about issues in recent decades, due to its fundamental role in many image processing applications, such as image coding, image synthesis, pattern recognition and so on. From the point of view of human vision, most images, including both synthetic ones and natural ones, can be easily segmented into several phases by human naked eyes. However, this human visual system has complex neurobiological criterion for segmentation that would be challenging to imitate exactly for a computer. In a machine vision system, the segmentation criterion is usually

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based on the difference in intensity value, regional statistics and other features of disjoint phases.

The general N-phase segmentation problem can be illustrated as follows: Given a gray scale image $I:\Omega\rightarrow\mathbb{R}$, where the image domain Ω is a bounded, smooth and open subset of $\mathbb{R}^2$, the aim is to partition Ω into N disjoint connected open subsets $\{\Omega_i\}_{i=1}^N$, i.e., $\Omega_i\bigcap\Omega_j=\emptyset$, $j\neq i$ and $\bigcup_{i=1}^N\Omega_i\bigcup\Gamma=\Omega$, by certain suitable measures, where Γ is the union of the part of boundaries of the Ω_i inside Ω. The segmentation is achieved by minimizing the summary energy functional:

$$\min\left\{E(\Gamma,\Omega)=\sum_{i=1}^N\Big(\int_{\Gamma_i}ds+\lambda\int_{\Omega_i}r_i(x)dx\Big)\right\},\tag{1.1}$$

where the first term is to restrict the boundary of the segment as short as possible and the second term is to use the error function $r_i(x)$ to measure the similarity between the features of the underlying x and of different phases, so as to determine which phase it should belong to. Many edge-based and region-based models in literature share this idea in nature, while differs in the representation of the region Ω_i, the error function r_i and some other regularization terms might be added.

Snakes/Active Contours [22] and Balloons [13] are classical edge-based segmentation methods which minimize the energy through deforming an initial contour towards the object boundaries. However this energy model cannot deal with topological changes. Incorporating the techniques of curve evolution and geometric flows, many active-contour-based models were proposed(e.g., see [5, 6, 24]). These models works for those images whose object boundaries are smooth and clearly defined by intensity gradient, but in many cases the boundaries might not be simply defined in such a way, especially for texture images. This difficulty inspires many researchers to integrate regional features with edge information. Chan and Vese [7] incorporated the classical Active Contour model with Mumford-Shah functional [35] and assumed each region can be approximated by a piece-wise constant function. The difference with the classical Active Contour model is that its stopping criterion does not rely on gradient of the image. Thus the blurry edges or gaps caused by missing edges would not be an issue. Later they extended this model to handle vector-valued images [8] and recently many other variances were proposed (see [17, 41–43]). M-S model which studied by Mumford and Shah [35] holds the idea that each sub-region Ω_i can be approximated by a piece-wise smooth function s_i. Hence their model consists of three terms, one is the error function $r_i=(I-s_i)^2$, the other two are regularizer on the approximate function $|\nabla s_i|^2$ and on the total length of the boundaries $|\Gamma|$. It has become one of the most dominating region-based models due to its great compatibility with other models and variability of the interpretation, for instance, Bresson et al. [3] incorporate the boundary information and shape prior to the M-S model, Brox and Cremers [44] introduced a statistical interpretation of the M-S functional, recently Sochen and Bar proposed a generalization of the original M-S model in combination of the Beltrami Framework.