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Acceleration Strategies Based on an Improved Bubble Packing Method

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Abstract. The bubble packing method can generate high-quality node sets in simple and complex domains. However, its efficiency remains to be improved. This study is a part of an ongoing effort to introduce several acceleration schemes to reduce the cost of simulation. Firstly, allow the viscosity coefficient c in the bubble governing equations to change according the coordinate of the bubble which are defined separately as odd and normal bubbles,and meanwhile with the saw-shape relationship with time or iterations. Then, in order to relieve the over crowded initial bubble placement, two coefficients w_1 and w_2 are introduced to modify the insertion criterion. The range of those two coefficients are discussed to be $w_1=1$, $w_2\in[0.5,0.8]$. Finally, a self-adaptive termination condition is logically set when the stable system equilibrium is achieved. Numerical examples illustrate that the computing cost can significantly decrease by roughly 80% via adopting various combination of proper schemes (except the uniform placement example), and the average qualities of corresponding Delaunay triangulation substantially exceed 0.9. It shows that those strategies are efficient and can generate a node set with high quality.

AMS subject classifications: 65L50, 65B99

Key words: Bubble packing method, algorithm efficiency, viscosity coefficient, mesh generation, Delaunay triangulation.

1 Introduction

Engineering analysis of mechanical systems have been addressed by deriving a partial differential equation systems through basic physical principles such as equilibrium, conservation of energy and mass, the laws of thermodynamics and Newton's laws of motion. However, once formulated, solving the resulting mathematical models is often intractable, or even impossible, especially when the resulting models are non-linear. Over

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the past few decades many such problems have emerged in various computer applications, and along with a corresponding collection of solution techniques. These applications include engineering analysis, computer graphics, layered manufacturing, surface design, and shape reconstruction [1].

In recent dozens of years, the finite element method (FEM) is the dominant discretization technique in solving such mechanical problems, while the basic concept in the physical interpretation of the FEM is the subdivision of the mathematical model into disjoint (non-overlapping) components of simple finite elements. For those numerical methods fall into the mesh-based category, the accuracy and convergence properties are dependent on the size and shape of the elements, which consequently have a positive influence on the distribution of the corresponding nodes set [2].

In this paper, attentions are fixed on analysing the node placement method called Bubble Packing Method (BPM), or bubble meshing developed by Shimada et al. [1,3–5], which is on account of the fact that a pattern of closely packed circles mimics a Voronoi diagram so that a set of well-shaped Delaunay triangles can be created by connecting the centers of the circles, this packed configuration is obtained by defining proximity-based interacting forces among the circles and finding a force-balancing configuration using dynamic simulation.

Since then, a fully automated quadrilateral meshing for finite element analysis was presented, in which the directionality of the mesh is precisely controlled [6–8]. Itoh and Shimada [9] presented a triangular-to-quadrilateral mesh conversion method that can control the directionality of the output quadrilateral mesh according to a user-specified vector field. Additionally, Yamakawa et al. [10] suggested an algorithm for anisotropic tetrahedral meshing.

Kim et al. (2003) [11] employed a modified octree technique to place initial bubbles in three-dimensional bodies, which leads to a drastic reduction of errors for each of the two- and three-dimensional domains, from the present bubble packing technique combined with the adaptive refinement based upon Zienkiewicz and Zhu error estimator [12, 13]. Chung et al. [14, 15] then proposed a new remeshing algorithm using the BPM in two-dimensional finite element analysis and applied this algorithm to the large deformation problem, it works well at the region with large error, it is further able to control the refinement area and the new mesh size easily through the maximum permissible relative error $\eta^*_{\max}$ and the bubble size control factor q. Nie et al. [2] verified the bubble system convergence by studying the changes of the average speed in the dynamic simulation, and also pointed out that physical parameters (e.g., damping coefficient) should be properly selected, which directly impact on the convergence speed of the bubble system. Moreover, parallelization for large-scale problems can also be easily proposed for reduction of the computational cost [16]. Rossi and Shimada et al. [17–19] presented a computerized planning scheme for prostate cryosurgery using a variable insertion depth strategy based on BPM in comparison with the experimentally applied cryoprobe layout, and it is commented that this method was associated with a high computational cost.

Recently, Wu et al. [20] employed BPM into simulating both square and polar lid-