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Numerical Study of Stability and Accuracy of the Immersed Boundary Method Coupled to the Lattice Boltzmann BGK Model

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Abstract. This paper aims to study the numerical features of a coupling scheme between the immersed boundary (IB) method and the lattice Boltzmann BGK (LBGK) model by four typical test problems: the relaxation of a circular membrane, the shearing flow induced by a moving fiber in the middle of a channel, the shearing flow near a non-slip rigid wall, and the circular Couette flow between two inversely rotating cylinders. The accuracy and robustness of the IB-LBGK coupling scheme, the performances of different discrete Dirac delta functions, the effect of iteration on the coupling scheme, the importance of the external forcing term treatment, the sensitivity of the coupling scheme to flow and boundary parameters, the velocity slip near non-slip rigid wall, and the origination of numerical instabilities are investigated in detail via the four test cases. It is found that the iteration in the coupling cycle can effectively improve stability, the introduction of a second-order forcing term in LBGK model is crucial, the discrete fiber segment length and the orientation of the fiber boundary obviously affect accuracy and stability, and the emergence of both temporal and spatial fluctuations of boundary parameters seems to be the indication of numerical instability. These elaborate results shed light on the nature of the coupling scheme and may benefit those who wish to use or improve the method.

AMS subject classifications: 76M28, 74F10

Key words: Immersed boundary method, lattice Boltzmann method, fluid-structure interaction, flexible boundary, complex boundary, accuracy, stability, verification.

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1 Introduction

The immersed boundary (IB) method is both a novel mathematical formulation and a numerical method for fluid-structure interaction (FSI). It is particularly suitable for FSI problems with flexible structures, while rigid boundaries can also be treated well. The first version of the IB method was proposed by C. Peskin [1] in 1972 for simulating the flow patterns around natural heart valves. It has become a general method for computer simulation of biological structures interacting with fluids [2]. Some representative applications of IB methods include: blood flow in the human heart [3, 4], FSI of natural and prosthetic cardiac valves [5, 6], aquatic animal locomotion [7], wave propagation in the cochlea [8], platelet aggregation during blood clotting [9], flow of suspensions [10], valveless pumping [11], flow and transport in renal arterioles [12], cell and tissue deformation under shear flow [13], insect flight [14], hemodynamics in the aorta [15], free swimmers in viscoelastic fluids [16], diffusion of integral membrane proteins [17], and dynamics of parachute opening [18].

Since Peskin's pioneering work, many modifications and refinements have been proposed to extend and improve the method. These include the immersed interface method which was an improvement to second-order accuracy for a neutrally-buoyant closed boundary [19], the blob-projection method which was an improvement to higher Reynolds numbers [20], the immersed continuum method which was extension to finite element formulation [21], and the immersed finite element method which was an extension to the compressible case using a finite element formulation [22]. Within the IB method itself, there exist quite a few different versions. These include the original versions [2],the volume conserved version [23], the adaptive mesh refinement version [24], the (formally) second-order versions [25, 45], the multigrid version [26], the penalty version [27], the implicit versions [28], and the lattice-Boltzmann (LB) version [29–35].

The lattice Boltzmann version of IB method has been undergoing a rapid development in recent years, partly because the LB method [52] is an efficient, relatively simple, and essentially parallel scheme for fluid flow simulations, and the IB-LB coupling has been proven to be effective for simulating fluid-structure interaction (FSI). Some works along this line include [29–39]. Feng et al. first published a coupled IB-LB scheme for simulating particle-fluid interaction problems [34]. Later, Peng upgraded the scheme by using a multi-block lattice and multi-relaxation-time LB scheme to enhance stability and to implement local grid refinement [35]. Shu improved the convergence of the coupling scheme by correcting the velocity to enforce the physical boundary conditions [37]. Dupuis simulated the flow past an impulsively started cylinder [38]. Niu improved the calculation of the boundary force on the fluid [39]. Peng carried out comparative study of IB-LB and LB bounce-back treatment of boundary [36]. Kang compared direct-forcing IB-LB methods for stationary complex boundaries [44]. The above works were intended for *rigid*-body-fluid interaction. On the other hand, for FSI problems with *flexible* boundary, Cheng proposed a scheme suitable for rapidly moving boundary and large pressure gradient [43], Hao proposed an implicit scheme to improve the robustness [41], Zhang