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A Modified Gas-Kinetic Scheme for Turbulent Flow

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Abstract. The implementation of a turbulent gas-kinetic scheme into a finite-volume RANS solver is put forward, with two turbulent quantities, kinetic energy and dissipation, supplied by an allied turbulence model. This paper shows a number of numerical simulations of flow cases including an interaction between a shock wave and a turbulent boundary layer, where the shock-turbulent boundary layer is captured in a much more convincing way than it normally is by conventional schemes based on the Navier-Stokes equations. In the gas-kinetic scheme, the modeling of turbulence is part of the numerical scheme, which adjusts as a function of the ratio of resolved to unresolved scales of motion. In so doing, the turbulent stress tensor is not constrained into a linear relation with the strain rate. Instead it is modeled on the basis of the analogy between particles and eddies, without any assumptions on the type of turbulence or flow class. Conventional schemes lack multiscale mechanisms: the ratio of unresolved to resolved scales – very much like a degree of rarefaction – is not taken into account even if it may grow to non-negligible values in flow regions such as shocklayers. It is precisely in these flow regions, that the turbulent gas-kinetic scheme seems to provide more accurate predictions than conventional schemes.

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1 Introduction

Turbulence is a multiscale problem: the challenge lies in either capturing all the scales of motion as is done in the DNS approach or modeling the unresolved scales of motion. Most of the turbulence models, in either RANS or LES, still rely on the idea of eddy viscosity – developed in analogy with the kinetic theory of gases. Over a century after its

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introduction, its drawbacks are well known and accepted [22]. From the kinetic point of view, the most relevant disadvantage remains the lack of a clear scale separation between resolved and unresolved scales of motion. In rarefied gas dynamics, the ratio of unresolved to resolved scales is used to assess the degree of rarefaction and when it reaches a non-negligible value, the validity of the Navier-Stokes set of equations is questioned. In the continuum regime, whenever turbulence is modeled, the degree of rarefaction is also influenced by unresolved turbulent fluctuations and may make the Navier-Stokes approach unsuitable.

Unlike Navier-Stokes schemes, numerical schemes developed on the basis of the Boltzmann equation dispose of "multiscale" mechanisms, i.e. they can adjust to the changing size of unresolved fluctuations. Among these approaches, one may recall the Lattice Boltzmann Method (LBM) and gas-kinetic schemes. The studies by Chen *et al.* [5, 6] have investigated turbulence modeling in the LBM: the present study exploits the findings in [5, 6] in order to explore turbulence modeling following the RANS approach with a gas-kinetic scheme. A well-validated gas-kinetic scheme [32], is adapted to the simulation of turbulence following the RANS approach; it does not propose a novel turbulence model nor a new modeling technique. Instead, a turbulent relaxation time is derived from the turbulent quantities provided by a standard two-equation turbulence model. The analysis of the resulting turbulent scheme shows that the turbulent stress tensor is corrected as a function of the degree of rarefaction, which may assume "transitional" values in shocklayers. The results of numerical experiments, based on different examples of shock-turbulent boundary layer interaction, confirm that the gas-kinetic scheme has the ability to do better than conventional schemes.

A number of gas-kinetic schemes have been developed over the latest 20 years [7, 19, 20, 32, 35] with the aim to achieve a physically more consistent mathematical model of fluid mechanics. Gas-kinetic schemes are invariably more accurate than conventional schemes, and might be able to resolve shock-layers. Besides, they are more suitable to high-order reconstruction [17,34,36] and may be used as a platform to investigate rarefied flow [18, 33].

The most evident characteristics of these schemes concerns the modeling of collisions. Whereas in conventional schemes, collisions are modeled by a viscosity and assuming a linear stress – strain relation, gas kinetic schemes use a relaxation time – related to collision frequency and hence to viscosity. The BGK model, depending on the scheme parameters, generates a number of different contributions: a stress tensor proportional to viscosity and strain rate, but also a number of non-linear correction terms, related to the local degree of rarefaction and the macroscopic gradients [20, 32]. These correction terms represent the kinetic effects that affect the transport properties of the flow, as soon as the degree of rarefaction becomes significant. In most of the flows of engineering and scientific interest, rarefaction may become significant only in shocklayers; however, even in the presence of shocks, conventional schemes do not correct advection: the solution of the Riemann problem is unaffected by the viscosity of the flow.

The numerical experiments proposed in this study address in particular the inter-