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Conditional Simulation of Flow in Heterogeneous Porous Media with the Probabilistic Collocation Method

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Abstract. A stochastic approach to conditional simulation of flow in randomly heterogeneous media is proposed with the combination of the Karhunen-Loeve expansion and the probabilistic collocation method (PCM). The conditional log hydraulic conductivity field is represented with the Karhunen-Loeve expansion, in terms of some deterministic functions and a set of independent Gaussian random variables. The propagation of uncertainty in the flow simulations is carried out through the PCM, which relies on the efficient polynomial chaos expansion used to represent the flow responses such as the hydraulic head. With the PCM, existing flow simulators can be employed for uncertainty quantification of flow in heterogeneous porous media when direct measurements of hydraulic conductivity are taken into consideration. With illustration of several numerical examples of groundwater flow, this study reveals that the proposed approach is able to accurately quantify uncertainty of the flow responses conditioning on hydraulic conductivity data, while the computational efforts are significantly reduced in comparison to the Monte Carlo simulations.

AMS subject classifications: 60H35, 65C50, 65M70, 76S05

Key words: Conditional simulation, probabilistic collocation method, Karhunen-Loeve expansion, polynomial chaos expansion.

1 Introduction

It is well recognized that the geological formations normally exhibit spatial heterogeneity to certain degrees. On the other hand, our information about the formations is limited due to insufficient measurements. As such, the properties of geological formations such as the hydraulic conductivity are usually considered as random space functions, and the

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equations describing flow and transport in the subsurface become stochastic. Extensive studies on flow and transport in random porous media have been conducted in the past and many stochastic approaches have been developed [4, 6, 7, 20, 22, 29].

Uncertainties of the subsurface flow can be quantified through stochastic simulation of flow in random porous media, in that the statistical moments or probability density functions can be evaluated for the flow responses of interests. When some measurements of the hydraulic conductivity are prescribed in the process of stochastic simulation, it is called the conditional simulation [5, 10, 17, 21]. The conditioning on the measured hydraulic conductivity can reduce the overall uncertainty of the hydraulic conductivity field, and that of responses of the flow and transport.

The most common approach to stochastic simulation of flow in porous media is the Monte Carlo method [1,29]. In this approach, a random field is represented by an ensemble of equally probable realizations. With each of the realizations as input, multiple flow simulations are performed independently, and statistical properties of flow responses can be evaluated. Monte Carlo method is straightforward to implement, either for unconditional or conditional simulation. However, it normally requires a large number of simulations to achieve statistical convergent results, thus it is computationally demanding, which prohibits its applications in large scale problems.

An alterative to the Monte Carlo method is the moment equation method, in which a system of deterministic differential equations governing the statistical moments (usually the first two) of the random variables are derived with the perturbation method or the closure approximation method [9, 11, 19, 27, 28, 31]. However, the number of resulting deterministic equations in the moment equation method is dependent on the number of grid blocks in the numerical simulation, thus the computational cost of the this method is still high especially for large scale problems [29]. And it is limited to relatively small variance of hydraulic conductivity. The Karhunen-Loeve decomposition based moment equation approach (KLME) was developed for unconditional simulation of single phase flow in porous media [17], which was applied for groundwater flow and transport problems [2,3,16,26]. Lu and Zhang [17] extended the KLME to conditional simulation of flow in heterogeneous media, to incorporate the existing measurements of hydraulic conductivity.

The probabilistic collocation method (PCM) is another efficient stochastic approach [13, 24]. It is based on the polynomial chaos expansions of random variables or fields, and a collocation technique is used to solve for the coefficients of the polynomial chaos expansions, which leads to uncoupled deterministic differential equations, similar to the governing flow equations. Li and Zhang [13] explored the PCM for single phase flow and showed its superiority compared to other stochastic approaches. With the PCM, a small number of flow simulations are performed independently, and the existing simulators can be employed. The method has been applied for efficient uncertainty quantification of unconfined groundwater flow [23], unsaturated and multiphase flows [14, 15]. However in these studies, the PCM was only used for unconditional simulation.

In this study, an approach for conditional simulation of flow in randomly heteroge-