Commun. Comput. Phys.
doi: 10.4208/cicp.090813.160414a

Vol. **16**, No. 4, pp. 1031-1055
October 2014

Hydrodynamic Interaction of Elastic Capsules in Bounded Shear Flow

D. V. Le[1] and Zhijun Tan[2,3,*]

[1] *Institute of High Performance Computing, Agency for Science, Technology and Research, 1 Fusionopolis Way, #16-16 Connexis, Singapore 138632.*
[2] *School of Mathematics and Computational Science, Sun Yat-sen University, Guangzhou 510275, China.*
[3] *Province Key Laboratory of Computational Science, Sun Yat-sen University, Guangzhou 510275, China.*

Received 9 August 2013; Accepted (in revised version) 16 April 2014

Communicated by Boo-Cheong Khoo

Available online 8 August 2014

Abstract. This paper presents a modified Loop's subdivision algorithm for studying the deformation of a single capsule, the hydrodynamic interaction between two capsules and the hydrodynamic diffusion of a suspension of capsules in bounded shear flow. A subdivision thin-shell model is employed to compute the forces generated on the surface of the elastic capsule during deformation. The capsule surface is approximated using the modified Loop's subdivision scheme which guarantees bounded curvature and C^1 continuity everywhere on the limit surface. The present numerical technique has been validated by studying the deformation of a spherical capsule in shear flow. Computations are also performed for a biconcave capsule over a wide range of shear rates and viscosity ratios to investigate its dynamics. In addition, the hydrodynamic interaction between two elastic capsules in bounded shear flow is studied. Depending on the wall separation distance, the trajectory-bifurcation points that separate reversing and crossing motions for both spherical and biconcave capsules can be found. Compared to the spherical capsules, the biconcave capsules exhibit additional types of interaction such as rotation and head-on collision. The head-on collision results in a large trajectory shift which contribute to the hydrodynamic diffusion of a suspension. A suspension of a large number of biconcave capsules in shear flow is also simulated to show the ability of the modified scheme in running a large-scale simulation over a long period of time.

AMS subject classifications: 65M06, 76D05, 74F10, 92C10

Key words: Thin shell, elastic capsules, red-blood cells, subdivision surfaces, immersed boundary method, Navier-Stokes equations.

*Corresponding author. *Email addresses:* `ledv@ihpc.a-star.edu.sg` (D. V. Le), `tzhij@mail.sysu.edu.cn` (Z. Tan)

1 Introduction

In recent years, the interest in particle motion has been motivated by applications in microfluidics and cell biomechanics. Numerous studies have been performed theoretically, computationally and experimentally for liquid droplets, elastic capsules and red blood cells [2, 11, 13, 36] to understand how their behaviors affect the flow characteristics. The motion of a single spherical and slender rigid particle has been well understood. The dynamics of an isolated elastic capsule or single cell in shear flow is also relatively well known. In a simple shear flow, spherical capsules exhibit a stationary tank-treading behavior [32] while spheroidal and biconcave capsules undergo swinging or tumbling motion [1]. The dynamics of the capsule depends on the viscosity ratio between the internal and suspending fluids, applied shear rate and the elastic properties of the capsule membrane. These parameters also affect the hydrodynamics interaction between two particles in shear flow.

The simplest of problems concerning the interaction between two particles has been considered for a pair of neutrally buoyant rigid spheres [3, 9, 17]. The analysis of Batchelor and Green [3] showed that, depending on the separation distance between the two spheres, they will either orbit around each other or passing each other before returning to their initial transverse positions. This result is in agreement with the experiment studied in [9] for the interaction of two rigid spheres in Couette flow. Recently, Pozrikidis [30, 31] developed accurate numerical methods for simulating the interception of two spherical particles with arbitrary radii in simple shear flow and discussed the particle self-diffusivities.

Unlike the interaction of two smooth rigid spheres, the hydrodynamic interaction between two liquid droplets results in an irreversible vertical displacement [5, 24] which leads to shear-induced diffusion in a suspension of liquid particles. Such irreversible shift in the particle trajectories has also been observed for liquid capsules [4, 10, 18, 19]. Depending on the initial separation distance, the two spherical capsules in shear flow either cross over each other or undergo reversing motion as observed in [10]. The reversing motion, in which the two capsules reversed their directions of motion upon approaching, is the result of the wall effect in the cross-flow direction. In this article, we investigate numerically the hydrodynamic interactions between two identical spherical capsules and red-blood cells in bounded shear flow and study the effects of channel heights on the reversing motion. We also investigate numerically the hydrodynamic interactions of a large number of biconcave capsules in a dense suspension to study the hydrodynamic diffusion or shear-induced diffusion. Hydrodynamic diffusion of capsules in a suspension is a phenomenon in which capsules exhibit diffusive motion during flow due to the interactions between them. This motion is responsible for phenomena such as blood viscosity changes in small capillaries and the natural propensity for blood cells to migrate to certain preferred locations within a conduit, which is an important application of this phenomenon. For example, taking advantage of the tendency for white blood cells to marginate in long channels, it is possible to design a microfluidic device for separating