A Local Velocity Grid Approach for BGK Equation

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Abstract. The solution of complex rarefied flows with the BGK equation and the Discrete Velocity Method (DVM) requires a large number of velocity grid points leading to significant computational costs. We propose an adaptive velocity grid approach exploiting the fact that locally in space, the distribution function is supported only by a sub-set of the global velocity grid. The velocity grid is adapted thanks to criteria based on local temperature, velocity and on the enforcement of mass conservation. Simulations in 1D and 2D are presented for different Knudsen numbers and compared to a global velocity grid BGK solution, showing the computational gain of the proposed approach.

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1 Introduction

In hydrodynamic regimes, fluid flows can be simulated thanks to standard models such as Navier-Stokes or compressible Euler equations. However, some regimes cannot be qualified as hydrodynamic and the continuum equations are not able to correctly describe the dynamics of the flow. The parameter that dictates whether or not a flow is hydrodynamic is the Knudsen number Kn. It is defined as the ratio between the mean free path λ between the particles and the characteristic length of the physical problem L. When this number goes towards zero, the hydrodynamic regime is reached. For large

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Knudsen numbers (usually higher than 10^{-2}), the regime is qualified as rarefied and the governing equation is the Boltzmann equation [9].

Directly solving the Boltzmann equation is computationally prohibitive because of the high-dimensionality and of the complexity of the collision operator, see for instance [14]. However, several numerical methods exist in the literature to get around this difficulty. One of the most popular methods is the Direct Simulation Monte Carlo (DSMC, [7]). This is a statistical approximation of the Boltzmann equation that is numerically viable for high Knudsen numbers, but becomes very costly for low Knudsen numbers as the number of collisions increases and a strong restriction appears on the time step [21]. For these regimes it also has the disadvantage of generating noisy results.

Hence, attempts have been made to derive numerical solvers for the Boltzmann equation which are not based on particles [25]. For a recent review of deterministic methods for Boltzmann equation, see [15], and the references therein. Numerical solvers for Boltzmann equation have to deal with the complexity of the collision term. For this reason, simplified kinetic models can be extremely attractive. A particularly successful model is the BGK model [6], in which the collision term of Boltzmann equation is approximated as a relaxation towards a Maxwellian distribution function. More recently, an extension to BGK has been widely studied to include thermal effects which are not correctly represented in the standard BGK model. It is the ES-BGK model [3]. The ability of this model to correctly approximate the full Boltzmann equation has been studied in [2].

For small relaxation times, a strong restriction on the time step exists also for explicit schemes to integrate the BGK model, but this constraint can be treated by using implicit schemes, such as IMEX [19,22] which have been successfully applied to the BGK model, [23] and [13]. Recently, this technique has been extended to the efficient integration of the ES-BGK model [1,17]. IMEX numerical schemes provide efficient solvers which give the correct asymptotic properties [16,18,24].

Such models are usually implemented thanks to the discrete velocity method (DVM) [8], that requires a discretization of the velocity space. The same global velocity grid is employed for each space discretization point. A major bottleneck of such an approach is the size of the velocity grid. Indeed, the boundaries of the velocity grid have to be determined according to the maximum velocity and temperature in the flow while its spacing depends on the minimum temperature. In realistic test cases like hypersonic atmosphere re-entries or satellite engines, very strong gradients in temperature can be observed. Then, the solution needs a very large and fine grid, so that the computational requirements increase quickly and make the phenomenon difficult to simulate especially in 3D.

This important problem has already been addressed in [11]. There, an AMR technique is presented in velocity space for the unified gas-kinetic scheme [27]. A careful work has been carried out on the error estimation due to quad/octree meshes to make the method accurate. Since the grids are not connected, interpolation is required in the transport step, to match the velocity grids in the neighbouring space cells. This interpolation can lead to approximation errors and possible increasing computational time. In [4] a criteria to