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A Unified Fractional-Step, Artificial Compressibility and Pressure-Projection Formulation for Solving the Incompressible Navier-Stokes Equations

László Könözsy[1,*] and Dimitris Drikakis[1]

[1] *Fluid Mechanics and Computational Science, Cranfield University, Cranfield, Bedfordshire, MK43 0AL, United Kingdom.*

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Abstract. This paper introduces a unified concept and algorithm for the fractional-step (FS), artificial compressibility (AC) and pressure-projection (PP) methods for solving the incompressible Navier-Stokes equations. The proposed FSAC-PP approach falls into the group of pseudo-time splitting high-resolution methods incorporating the characteristics-based (CB) Godunov-type treatment of convective terms with PP methods. Due to the fact that the CB Godunov-type methods are applicable directly to the hyperbolic AC formulation and not to the elliptical FS-PP (split) methods, thus the straightforward coupling of CB Godunov-type schemes with PP methods is not possible. Therefore, the proposed FSAC-PP approach unifies the fully-explicit AC and semi-implicit FS-PP methods of Chorin including a PP step in the dual-time stepping procedure to a) overcome the numerical stiffness of the classical AC approach at (very) low and moderate Reynolds numbers, b) incorporate the accuracy and convergence properties of CB Godunov-type schemes with PP methods, and c) further improve the stability and efficiency of the AC method for steady and unsteady flow problems. The FSAC-PP method has also been coupled with a non-linear, full-multigrid and full-approximation storage (FMG-FAS) technique to further increase the efficiency of the solution. For validating the proposed FSAC-PP method, computational examples are presented for benchmark problems. The overall results show that the unified FSAC-PP approach is an efficient algorithm for solving incompressible flow problems.

AMS subject classifications: 76D05, 65M08, 65B99, 65Y20

Key words: Navier-Stokes equations, characteristics-based Godunov-type scheme, unified method.

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*Corresponding author. *Email addresses:* `laszlo.konozsy@cranfield.ac.uk` (L. Könözsy), `d.drikakis@cranfield.ac.uk` (D. Drikakis)

1 Introduction

This paper introduces a unified solution concept of the fractional-step (FS), artificial compressibility (AC) and pressure-projection (PP) methods for solving the incompressible Navier-Stokes equations. The development of the unified FSAC-PP method [1] is carried out within the framework of Godunov-type methods [1,2]. Due to the fact that the characteristics-based (CB) Godunov-type methods are applicable directly to the hyperbolic system of the AC formulation and not to the elliptical FS-PP (split) methods, thus the straightforward coupling of CB Godunov-type schemes with PP methods is not possible. To take advantage of the accuracy and convergence properties of CB Godunov-type schemes, and increase the stability and efficiency of the classical AC method at (very) low and moderate Reynolds numbers, the FSAC-PP approach is proposed.

The AC method was developed by Chorin [3] introducing a perturbed continuity equation based on a pseudo-time derivative for the pressure. This equation has no physical meaning until the steady-state solution is achieved [4]. The AC method has good numerical features for stationary flows, but for unsteady and/or low Reynolds number flows, the stability condition of the dual-time stepping procedure and the choice of the AC parameter can become too restrictive, thus leading to slow convergence rates [4].

For solving unsteady, incompressible flows, Chorin [5] and Temam [6] introduced the FS-PP method based on the orthogonality theorem of Ladyzhenskaya [7], which is also known as the Helmholtz-Hodge or Hodge decomposition [3,5,8–11]. According to this theorem, a vector field can be decomposed into a solenoidal (divergence-free) part and an irrotational part. The first FS estimates an intermediate velocity field neglecting the pressure gradient term from the momentum equation. The second FS projects the predicted intermediate velocity field into a divergence-free (exact projection), or numerically nearly divergence-free (approximate projection) vector field relying on the solution of a pressure-Poisson equation. Kim and Moin [12] proposed an application of the FS-PP method to the solution of the three-dimensional, time-dependent incompressible Navier-Stokes equations. Perot [13] accomplished a detailed analysis on the FS-PP method to overcome the first-order temporal accuracy of the flow field solution by using a generalized block LU decomposition of the governing equations for the primitive variables.

In the last 20 years, researchers have also made efforts to combine various high-resolution schemes with FS-PP methods for single- and variable-density flows [8, 14]. Bell et al. [8] developed a second-order, exact projection method in conjunction with Godunov-type methods [15] for the unsteady, incompressible Navier-Stokes equations. For variable-density incompressible flows, Bell and Marcus [9] established a second-order projection method, and Almgren et al. [10] introduced a conservative and adaptive projection method. Eberle [16] developed a CB scheme method for the three-dimensional compressible Euler equations, and the scheme was further extended by Drikakis et al. [17] for solving the incompressible Navier-Stokes equations. This scheme was also extended to three-dimensional incompressible flows [18, 19] in conjunction with a nonlinear, FMG-FAS algorithm, as well as to variable density flows [20–22]. Recently, Za-