Chaotic Driven Tunneling in Rectangular Double-Well

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Abstract. We consider a charged particle confined in a one-dimensional rectangular double-well potential, driven by an external periodic excitation at frequency Ω and with amplitude A. We find that there is the regime of the parametric resonance due to the modulation of the amplitude A at the frequency ω_{prm}, which results in the change in the population dynamics of the energy levels. The analysis relies on the Dirac system of Hamiltonian equations that are equivalent to the Schrödinger equation. Considering a finite dimensional approximation to the Dirac system, we construct the foliation of its phase space by subsets $\mathcal{F}_{ab}$ given by constraints $a \leq N_0 \leq b$ on the occupation probabilities N_0 of the ground state, and describe the tunneling by frequencies ν_{ab} of the system's visiting subsets $\mathcal{F}_{ab}$. The frequencies ν_{ab} determine the probability density and thus the Shannon entropy, which has the maximum value at the resonant frequency $\omega = \omega_{prm}$. The reconstruction of the state-space of the system's dynamics with the help of the Shaw-Takens method indicates that the quasi-periodic motion breaks down at the resonant value ω_{prm}.

Key words: Tunneling; double-well; parametric resonance.

1 Introduction: Tunneling in the finite dimensional Hamiltonian approximation

Driven transitions in a double well are instrumental for studying the tunneling in various fields of physics and chemistry, [1, 2]. Considerable attention has been drawn to the tunneling dynamics in the presence of a driving force with a time dependent amplitude. In his seminal paper [3], M. Holthaus showed that shaping the driving force may be instrumental in controlling the tunneling in a bistable potential. In particular, it was shown, [3], that choosing an appropriate envelope for laser pulse, one may perform the population transfer on time scales much shorter than the

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base tunneling time. This situation is intimately related to the problem of quantum chaos, which is generally approached within the framework of the quasi-classical approximation and Gutzwiller's theory. In fact, classical chaotic systems are often used as a clue to the quantum ones. In contrast, it would be very interesting to look at the quantum chaos the other way round and consider systems which need studying without approximations that could have bearing upon classical mechanics, for example, particles confined to potentials of a size comparable with the de Broglie wave length. This has also an additional interest owing to the fact that calculations within the framework of semiclassical theory should depart from the quantum ones on the time scale of $\hbar/\Delta E$, ΔE being the typical spacing between energy levels. Specifically, the Schrödinger equation describing the problem needs numerical studying.

The current approach to the tunneling generally focuses on localization of wave packets, and uses the concept of dwell time. The latter is usually taken in the form, [4,5],

$$\tau_D(a,b) = \int_{-\infty}^{+\infty} dt \int_a^b |\psi(x,t)|^2 \, dx. \tag{1.1}$$

Even though the concept of dwell time is generally recognized as an important characteristic of the wave packet, [6], it brings forward conceptual difficulties owing to the special role played by time in quantum mechanics, [6,7]. Consequently, it is often difficult to determine a quantity that should be both correct and practical for studying the tunneling dynamics in a double well potential, if we wish to describe the particle's localization in one of the potential wells. Fortunately, in certain cases the choice of the well really means the choice of an energy state, as illustrated in Fig. 1, and consequently there is an opportunity for considering the tunneling dynamics in the energy representation, instead of the x-one. To put it in a quantitative form, let us consider the characteristic function $\chi_{ab}(x)$ which is 1 if $a \leq x \leq b$ and 0 otherwise, and introduce the quantity

$$\nu_\phi(a,b) = \lim_{T \to \infty} \left\{ \frac{1}{T} \int_{-T/2}^{+T/2} \chi_{ab}(N_\phi) \, dt \right\}, \qquad N_\phi = |<\psi|\phi>|^2, \tag{1.2}$$

where $\psi(t)$ is a solution of the Schrödinger equation, and ϕ is a state of the system.

The time averaged probability $\nu_\phi(a,b)$ can be visualized as the frequency of visiting a region of states determined by constraint $a \leq N_\phi \leq b$. It is easy to see that (1.2) is similar to (1.1), $\nu_\phi(a,b)$ describing the system's dwell time in segment (a,b). At first sight, the calculation of $\nu_\phi(a,b)$ looks rather complicated, and in fact the direct numerical integration of the Schrödinger equation is time consuming and requires special precautions to overcome possible artifacts. However, to solve the problem one may use the method proposed by Dirac [8] for studying the interaction of an atomic system with radiation. It relies on the decomposition of the system's Hamiltonian

$$H = H_0 + V,$$

where H_0 is the main term, and V is the term describing the interaction with external field. If