

## Numerical Simulation of Airfoil Vibrations Induced by Turbulent Flow

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**Abstract.** The subject of the paper is the numerical simulation of the interaction of two-dimensional incompressible viscous flow and a vibrating airfoil with large amplitudes. The airfoil with three degrees of freedom performs rotation around an elastic axis, oscillations in the vertical direction and rotation of a flap. The numerical simulation consists of the finite element solution of the Reynolds averaged Navier-Stokes equations combined with Spalart-Allmaras or  $k-\omega$  turbulence models, coupled with a system of nonlinear ordinary differential equations describing the airfoil motion with consideration of large amplitudes. The time-dependent computational domain and approximation on a moving grid are treated by the Arbitrary Lagrangian-Eulerian formulation of the flow equations. Due to large values of the involved Reynolds numbers an application of a suitable stabilization of the finite element discretization is employed. The developed method is used for the computation of flow-induced oscillations of the airfoil near the flutter instability, when the displacements of the airfoil are large, up to  $\pm 40$  degrees in rotation. The paper contains the comparison of the numerical results obtained by both turbulence models.

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## 1 Introduction

The interaction of flowing fluids and vibrating structures is the main subject of aeroelasticity, which studies the influence of aerodynamic forces on an elastic structure. The

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flow-induced vibrations may affect negatively the operation and stability of aircrafts, blade machines, bridges, and many other structures in mechanical or civil engineering. The main goal of aero-elasticity is the prediction of the bounds of the structure stability, to cure the aero-elastic instabilities leading to flutter or divergence and to analyze post-critical regimes. This discipline is highly developed, particularly from engineering point of view (see, e.g., the monographs [10] and [34]).

From the point of view of mathematical theory, there are not too many works dealing with such problems, due to a high mathematical complexity of the problem, caused by the time-dependence of the domain occupied by the fluid and coupling of the system of equations describing flow and elastic structure. The mathematical simulation of fluid and structure interaction requires to consider viscous, usually turbulent flow, changes of the flow domain in time, nonlinear behaviour of the elastic structure and to solve simultaneously the evolution systems for the fluid flow and for the oscillating structure. Considering the Reynolds averaged Navier-Stokes equations and a vibrating structure with large displacements, the change of the fluid domain cannot be neglected. The methods with moving meshes [13, 25] must be employed and the application of efficient and robust methods for the numerical solution is required.

The subject of our attention is the numerical analysis of the interaction of viscous turbulent flow with a vibrating airfoil. Recent studies on numerical modelling of the postflutter behaviour of airfoils in laminar two-dimensional (2D) incompressible flow were overviewed by the authors in the previous study (Feistauer et al. [14]), where the method allowing the solution of large amplitude flow-induced vibrations of an airfoil with 3 degrees of freedom (3-DOF) was developed for laminar flow. However, none of the studies mentioned in this paper deals with turbulent flow, which is necessary to take into account for high Reynolds numbers ( $10^5 - 10^8$ ).

For an extensive treatment of turbulent flows, one can be referred, e.g. to [27, 40, 42, 43, 46]. Turbulent flow has a three-dimensional character, but in a number of cases, two-dimensional models are applied to the numerical simulation of turbulent flow. Similar situation appears in theory, as can be found in [15]. In a turbulent flow simulation, techniques based on the Reynolds averaged Navier-Stokes (RANS) equations are often applied. As a result, the system called Reynolds equations (see [40, Chapter 4]) is obtained. It contains the so-called Reynolds stresses, evaluated with the aid of a turbulent viscosity model. It can be computed from algebraic relations or it can be obtained with the aid of the solution of additional equations for turbulence quantities, such as  $k$  and  $\omega$  (see, e.g. [40, Chapter 10]).

The effect of turbulence in aeroelastic computations is studied in civil engineering as well as in turbomachine, nuclear and aerospace engineering applications. For example, Baxevanou et al. [2] modeled the aeroelastic stability of a wind turbine blade section. The Reynolds averaged Navier-Stokes equations for 2D incompressible flow were solved numerically using the finite volume method on structured, curvilinear grids using two versions of the  $k-\omega$  high Reynolds number model of Wilcox with wall functions and wall treatment.