Commun. Comput. Phys.
doi: 10.4208/cicp.021013.260614a

Vol. **17**, No. 1, pp. 259-286
January 2015

Multilevel Markov Chain Monte Carlo Method for High-Contrast Single-Phase Flow Problems

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Received 2 October 2013; Accepted (in revised version) 26 June 2014

Abstract. In this paper we propose a general framework for the uncertainty quantification of quantities of interest for high-contrast single-phase flow problems. It is based on the generalized multiscale finite element method (GMsFEM) and multilevel Monte Carlo (MLMC) methods. The former provides a hierarchy of approximations of different resolution, whereas the latter gives an efficient way to estimate quantities of interest using samples on different levels. The number of basis functions in the online GMsFEM stage can be varied to determine the solution resolution and the computational cost, and to efficiently generate samples at different levels. In particular, it is cheap to generate samples on coarse grids but with low resolution, and it is expensive to generate samples on fine grids with high accuracy. By suitably choosing the number of samples at different levels, one can leverage the expensive computation in larger fine-grid spaces toward smaller coarse-grid spaces, while retaining the accuracy of the final Monte Carlo estimate. Further, we describe a multilevel Markov chain Monte Carlo method, which sequentially screens the proposal with different levels of approximations and reduces the number of evaluations required on fine grids, while combining the samples at different levels to arrive at an accurate estimate. The framework seamlessly integrates the multiscale features of the GMsFEM with the multilevel feature of the MLMC methods following the work in [26], and our numerical experiments illustrate its efficiency and accuracy in comparison with standard Monte Carlo estimates.

AMS subject classifications: 65N30, 76M10

Key words: Generalized multiscale finite element method, multilevel Monte Carlo method, multilevel Markov chain Monte Carlo, uncertainty quantification.

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1 Introduction

Uncertainties in the description of reservoir lithofacies, porosity and permeability are major contributors to the uncertainties in reservoir performance forecasting. The uncertainties can be reduced by integrating additional data, especially dynamic ones such as pressure or production data, in subsurface modeling. The incorporation of all available data is essential for the reliable prediction of subsurface properties. The Bayesian approach provides a principled framework for combining the prior knowledge with dynamic data in order to make predictions on quantities of interest [24]. However, it poses significant computational challenges largely due to the fact that exploration of the posterior distribution requires a large number of forward simulations. High-contrast flow is a particular example, where the forward model is multiscale in nature and only a limited number of forward simulations can be carried out before becoming prohibitively expensive. In this paper, we present a framework for uncertainty quantification of quantities of interest based on the generalized multiscale finite element method (GMsFEM) and multilevel Monte Carlo (MLMC) methods. The GMsFEM provides a hierarchy of approximations to the solution, and the MLMC provides an efficient way to estimate quantities of interest using samples on multiple levels. Therefore, the framework naturally integrates the multilevel feature of the MLMC with the multiscale nature of the high-contrast flow problem.

Multiscale methods represent a class of coarse-grid solution techniques that have garnered much attention over the past two decades (see, e.g., [1, 2, 16, 22, 23, 25]). They all hinge on the construction of a coarse solution space that is spanned by a set of multiscale basis functions. In this paper we follow the framework of the Multiscale Finite Element Method (MsFEM) [22] in which the basis functions are independently pre-computed, and are obtained through solving a set of local problems that mimic the global operator, in the hope of capturing the fine scale behavior of the global system. Then a global formulation is used to construct a reduced-order solution. While standard multiscale methods have proven very effective for a variety of applications [13, 14, 16, 20], recent work has offered a generalized framework for enriching coarse solution spaces in case of parameter-dependent problems, where the parameter reflects the uncertainties of the system. Specifically, the GMsFEM is a robust solution technique in which the standard solution spaces from the MsFEM may be systematically enriched to further capture the fine behavior of the fine grid solution [3,11,12]. The additional basis functions are chosen based on localized eigenvalue problems.

The GMsFEM achieves efficiency via coarse space enrichment, which is split into two stages, following an offline-online procedure (see also [6,8,28,30]). At the first stage of the computation, a larger-dimensional (relative to the online space) parameter-independent offline space is formed. The offline space accounts for a suitable range of parameter values that may be used in the online stage, and constitutes a one-time preprocessing step. The offline space is created by first generating a set of "snapshots" in which a number of localized problems are solved on each coarse subdomain for a number of parameter