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An Implicit Algorithm for High-Order DG/FV Schemes for Compressible Flows on 2D Arbitrary Grids

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Abstract. A Newton/LU-SGS (lower-upper symmetric Gauss-Seidel) iteration implicit method was developed to solve two-dimensional Euler and Navier-Stokes equations by the DG/FV hybrid schemes on arbitrary grids. The Newton iteration was employed to solve the nonlinear system, while the linear system was solved with LU-SGS iteration. The effect of several parameters in the implicit scheme, such as the CFL number, the Newton sub-iteration steps, and the update frequency of Jacobian matrix, was investigated to evaluate the performance of convergence history. Several typical test cases were simulated, and compared with the traditional explicit Runge-Kutta (RK) scheme. Firstly the Couette flow was tested to validate the order of accuracy of the present DG/FV hybrid schemes. Then a subsonic inviscid flow over a bump in a channel was simulated and the effect of parameters was investigated also. Finally, the implicit algorithm was applied to simulate a subsonic inviscid flow over a circular cylinder and the viscous flow in a square cavity. The numerical results demonstrated that the present implicit scheme can accelerate the convergence history efficiently. Choosing proper parameters would improve the efficiency of the implicit scheme. Moreover, in the same framework, the DG/FV hybrid schemes are more efficient than the same order DG schemes.

AMS subject classifications: 65C20, 65L60, 65M08, 65M60, 68U20, 76M10, 76M12

Key words: Implicit algorithm, discontinuous Galerkin method, DG/FV hybrid Scheme, Newton iteration, Gauss-Seidel iteration.

1 Introduction

The use of unstructured meshes for computational fluid dynamics problems has become widespread due to their ability to discretize arbitrarily complex geometries and the ease

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of adaptation in enhancing the solution accuracy and efficiency through the use of adaptive refinement techniques. In recent years, significant progress has been made in developing numerical algorithms for the solution of the Euler and Navier-Stokes equations on unstructured grids. Nearly all production flow solvers are based on second-order numerical methods, either finite volume method (FVM), finite difference method (FDM) or finite element method (FEM). Nevertheless, many types of problems, such as computational aeroacoustics (CAA), vortex-dominant flows and large eddy simulation (LES) of turbulent flows, call for higher order accuracy (third-order and higher). The main deficiency of widely available, second-order methods for the accurate simulations of the above-mentioned flows is the excessive numerical diffusion and dissipation of vorticity. Applications of higher-order accurate, low-diffusion and low dissipation numerical methods can significantly alleviate this deficiency of the traditional second-order methods, and improve predictions of vortical and other complex, separated, unsteady flows. Therefore, various higher-order methods have been developed in the last two decades [1–5], especially for unstructured grids, including the well-known discontinuous Galerkin (DG) method [3–7], the spectral volume (SV) method [8–11], and the spectral difference (SD) method [12–14]. Interested readers can refer to the comprehensive review articles for higher-order methods by Ekaterinaris [15] and Wang [16].

As the leader of higher-order numerical methods for compressible flow computations in aerospace applications, the DG method has recently become popular for problems with both complex physics and geometry. The DG method was originally developed by Reed and Hill to solve the neutron transport equation [3]. The development of higher-order DG methods for hyperbolic conservation laws was pioneered by Cockburn, Shu and their collaborators in a series of papers on the Runge-Kutta DG (RKDG) method [4–7]. Many other researchers have made significant contributions in the development of the DG methods. Refer to [17] for a comprehensive review on the DG method history and literature.

However, the DG method does have a number of weaknesses, including the huge memory requirement and high computational cost. In order to improve the efficiency in both memory and CPU time for 3D realistic complex configurations, many hybrid approaches have been proposed, including 1) different schemes for inviscid and viscous flux discretization [18, 19]; 2) hybrid approach based on domain decomposition [20]; 3) hybrid approach based on local polynomial reconstruction, such as the $P_N P_M$ schemes [21–23], the reconstructed-based DG (RDG) scheme [24,25], the Hermite WENO (HWENO) schemes [26, 27].

In our previous work [28–30], by comparing the DG methods, the k-exact FV methods and the lift collocation penalty (LCP) methods [31,32], a concept of "static reconstruction" and "dynamic reconstruction" was introduced for higher-order numerical methods. Based on this concept, a class of high order DG/FV hybrid schemes was presented for 1D and 2D conservation law using a "hybrid reconstruction" approach. In the DG/FV hybrid schemes, the lower-order derivatives of the piecewise polynomial are computed locally in a cell by the DGM based on Taylor basis functions [33] (called as "dynamic