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REVIEW ARTICLE

Stochastic Collocation on Unstructured Multivariate Meshes

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Abstract. Collocation has become a standard tool for approximation of parameterized systems in the uncertainty quantification (UQ) community. Techniques for least-squares regularization, compressive sampling recovery, and interpolatory reconstruction are becoming standard tools used in a variety of applications. Selection of a collocation mesh is frequently a challenge, but methods that construct geometrically *unstructured* collocation meshes have shown great potential due to attractive theoretical properties and direct, simple generation and implementation. We investigate properties of these meshes, presenting stability and accuracy results that can be used as guides for generating stochastic collocation grids in multiple dimensions.

AMS subject classifications: 6502, 65C20

Key words: Stochastic collocation, unstructured methes, least-squares, compressive sampling, least interpolation.

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1 Introduction

The field of uncertainty quantification has enjoyed much attention in recent years as theoreticians and practitioners have tackled problems in the diverse areas of stochastic analysis, exascale applied computing, high-dimensional approximation, and Bayesian learning. The advent of high-performance computing has led to an increasing demand for efficiency and accuracy in predictive capabilities in computational models.

One of the persistent problems in Uncertainty Quantification (UQ) focuses on parameterized approximation to differential systems: Let u be a state variable for a system that is the solution to a physical model

$$\mathcal{L}(u;t,x,\omega)=0, \tag{1.1}$$

Above $x\in\mathbb{R}^p$ for $p=1,2,3$ is a spatial variable, $t\in\mathbb{R}$ is a temporal variable, and $\omega\in\Omega$ is a probabilistic event that encodes randomness on a complete probability space $(\Omega,\mathcal{F},\mathcal{P})$. We assume that the model (1.1) defines a map $\omega\mapsto u(t,x,\omega)$ with $u:\Omega\to B$ that is well-posed almost surely for some appropriate space B of (x,t)-dependent functions.

The operator $\mathcal{L}$ may represent any mathematical model of interest; examples that are popular in modern applied communities are elliptic partial differential equations, systems of time-dependent differential equations, parametric inverse problems, and data-driven optimization; e.g., [1, 17, 37, 57, 62, 69, 71]. The system defined by the operator $\mathcal{L}$ may include boundary value constraints, initial value prescriptions, physical domain variability, or any combination of these [81, 88, 89, 93].

The sought system response $u(t,x,\omega)$ is random/stochastic, given by the solution to (1.1). The stochastic dependence in (1.1) given by the event ω is frequently approximated by a d-dimensional random variable $Z(\omega)$. In some cases this parameterization of randomness is straightforward: e.g., in a Bayesian framework when ignorance about the true value of a vector of parameters is modeled by treating this parameter set as a random vector in (1.1). In contrast, it is common in models for an infinite-dimensional random field to contribute to the stochasticity, and in these cases parameterization is frequently accomplished by some finite-dimensional truncation procedure, e.g., via the Karhunen-Loeve expansion [44, 90], and this reduces the stochastic dependence in (1.1) to dependence on a random vector Z. In either case, a modeler usually wants to take $d\triangleq\dim Z$ as large as possible to encode more of the random variability in the model.

Under an assumption of model validity, the larger the stochastic truncation dimension d, the more accurate the resulting approximation. (Even when model validity is suspect, one can devise metamodeling procedures to capture model form error [50].) Therefore, it is mathematically desirable to take d as large as possible. We rewrite (1.1) to emphasize dependence on the d-dimensional random variable Z:

$$\mathcal{L}(u;t,x,Z)=0. \tag{1.2}$$

In this article we are ultimately interested in approximating $u(x,t,Z)$ or some functional