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Modeling 3D Magma Dynamics Using a Discontinuous Galerkin Method

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Abstract. Discontinuous Galerkin (DG) and matrix-free finite element methods with a novel projective pressure estimation are combined to enable the numerical modeling of magma dynamics in 2D and 3D using the library deal.II. The physical model is an advection-reaction type system consisting of two hyperbolic equations to evolve porosity and soluble mineral abundance at local chemical equilibrium and one elliptic equation to recover global pressure. A combination of a discontinuous Galerkin method for the advection equations and a finite element method for the elliptic equation provide a robust and efficient solution to the channel regime problems of the physical system in 3D. A projective and adaptively applied pressure estimation is employed to significantly reduce the computational wall time without impacting the overall physical reliability in the modeling of important features of melt segregation, such as melt channel bifurcation in 2D and 3D time dependent simulations.

AMS subject classifications: 65Z05, 65M22, 65M60

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1 Introduction

Generation and segregation of magma in the Earth and the interior of large planets has been a subject of intensive study in the Earth science community. Magma or melt can be

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generated by adiabatic decompressional melting during upwelling in the Earth's upper mantle and segregation of melt involves two-phase flow in which low viscosity melt percolates through a much more viscous solid matrix [4]. During its upward migration, melt generated in the deeper part of the upwelling mantle will interact both thermally and chemically with the overlying mantle. Our understanding of magma generation and segregation is constrained by physical evidence based in part upon geological field observations of ophiolites, where sections of the Earth's mantle have been uplifted and exposed at the surface. The key observation from these outcrops is the local depletion of a soluble mineral, orthopyroxene (opx). The elongated opx-depleted region, referred to as dunite in the geological literature, can be formed through a reactive infiltration instability, whereby highly porous regions form due to a positive feedback between melt percolation and dissolution of the soluble mineral opx in the solid. The reactive infiltration instability can form during magma migration in the mantle even when the percolating melt and the partially molten mantle is in local chemical equilibrium at a length scale of the mineral grain size ($1\sim 10s$mm) [12]. This is because the solubility of opx in the percolating melt, which is generated at a greater depth, increases with the decrease of depth in the Earth's mantle. It has been suggested that high-permeability dunite channels act as conduits for focused flow, whereby melt may efficiently segregate from its source region while still maintaining its geochemical signature developed at depth. For further discussion we refer to [9, 10] and the references therein.

Previous numerical studies utilizing a low-order finite difference scheme was presented in [18], and demonstrated the localization of the melt flow into high porosity channels. However, this work does not explicitly account for the soluble mineral opx, whose presence is essential to the formation of dunite channels, nor does it consider the effects of upwelling. Further numerical studies incorporating upwelling of the mantle was presented in [17], although this work does not discuss the specifics of the numerical model used. In [15], a high-order accurate numerical model was presented based on the physical models presented in [7,16], which include upwelling, a porosity-dependent bulk viscosity, and an additional equation to track the opx abundance. These two studies also assume a formulation of local chemical equilibrium with negligible diffusion in the melt. Here high-porosity channel forms along an opx solibility gradient in the flow direction. Linear stability analysis of this system predicts the emergence of compaction-dissolution waves [7]. In addition to the channeling instability, these features present a formidable challenge for numerical modeling. This challenge was previously addressed by developing a high-order numerical method that provides accurate resolution (i.e. simulations which can capture the channel splitting characteristics of melt fraction) at a reasonable cost. These numerical methods used a high-order discretization consisting of a tensor product of the Fourier collocation [15] or a high-order finite difference scheme [13] in the horizontal direction and discontinuous Galerkin methods in the vertical direction.

However, all previous works have employed a two-dimensional model and an expansion to the three dimensional problem would require substantial effort. Rather than attempting this, we explore the use of existing libraries and, in particular, the software li-