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A Model of a Quantum Particle in a Quantum Environment: A Numerical Study

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Abstract. We define and investigate, via numerical analysis, a one dimensional toy-model of a cloud chamber. An energetic quantum particle, whose initial state is a superposition of two identical wave packets with opposite average momentum, interacts during its evolution and exchanges (small amounts of) energy with an array of localized spins. Triggered by the interaction with the environment, the initial superposition state turns into an incoherent sum of two states describing the following situation: or the particle is going to the left and a large number of spins on the left side changed their states, or the same is happening on the right side. This evolution is reminiscent of what happens in a cloud chamber where a quantum particle, emitted as a spherical wave by a radioactive source, marks its passage inside a supersaturated vapour-chamber in the form of a sequence of small liquid bubbles arranging themselves around a possible classical trajectory of the particle.

AMS subject classifications: 35Q41, 65Zxx, 65N06, 65M06, 81S22, 68U20

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1 Introduction

In this paper we investigate numerically the dynamics of a quantum particle interacting with a quantum environment. More precisely, we consider the semi-classical limit regime

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of the dynamics. With this we mean that the average initial kinetic energy of the particle is assumed to be very large with respect to the energy exchanged by the particle with the environment.

The paradigmatic physical system we have in mind is the Wilson cloud chamber, the prototype of a tracking chamber for elementary particle detection [6, 13–15]. Inside the chamber, a very energetic α-particle, emitted in a radially symmetric way by a radioactive source, ionizes atoms of a super-saturated vapor. In turn, the ionized atoms become condensation nuclei, triggering the formation of sequences of liquid drops. The tracks one observes in real experiments look quite explicitly as classical particle trajectories.

In the early days of quantum mechanics Darwin, Heisenberg and Mott were the first to point out the seemingly paradoxical circumstance of an initial radially symmetric quantum state evolving into wave packets concentrated around classical trajectories. In different ways, they suggested that the problem could be faced taking into consideration that the wave function does not evolve in real space, but rather in the configuration space of the entire quantum system. This somehow obvious but extremely far-reaching intuition, exploited by Mott in his seminal work [21], remained unnoticed for decades.

More recently, researchers analyzed the cloud chamber problem focusing on different aspects of the interaction of a microscopic quantum system with a macroscopic one.

- **Decoherence:** the initial state of the α-particle can be seen as a superposition of coherent states each one having a well localized momentum direction. The superposition is initially strongly coherent: in absence of any interaction with the environment, two coherent states might interfere in a double-slit experiment. On the other hand, coherent states heading in different directions generate macroscopic ionization in different regions of the environment. Due to this particle-environment interaction, the state of the whole system evolves into an incoherent superposition of states supported in distant regions of the environment configuration space, in such a way that interference effects become negligible. For a general presentation of the decoherence phenomenon see e.g. [17] and references therein. For details on collisional decoherence in a tracking chamber we refer to the book [12].

- **Non demolition measure:** a microscopic system (e.g. a quantum particle) is said to undergo a non demolition measure if there is a basis of its states which are left unchanged by the interaction with the probe (the measurement apparatus). At the same time, in order to work as a measurement apparatus, the probe should evolve in different final states for different particle states in the basis. Some authors analyzed recently the effect of repeated non demolition measures and its relation with the "collapse" of the wave function in a quantum measurement [2]. In this language, the process described above can be rephrased in the following way: each coherent state with a well defined momentum direction is an element of the basis. For a very high average initial energy (semi-classical conditions) each state of the basis evolves almost freely in each weakly-inelastic scattering process, whereas the environment reacts in different ways according to the average momentum direction