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A Static Condensation Reduced Basis Element Approach for the Reynolds Lubrication Equation

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Abstract. In this paper, we propose a Static Condensation Reduced Basis Element (SCRBE) approach for the Reynolds Lubrication Equation (RLE). The SCRBE method is a computational tool that allows to efficiently analyze parametrized structures which can be decomposed into a large number of similar components. Here, we extend the methodology to allow for a more general domain decomposition, a typical example being a checkerboard-pattern assembled from similar components. To this end, we extend the formulation and associated *a posteriori* error bound procedure. Our motivation comes from the analysis of the pressure distribution in plain journal bearings governed by the RLE. However, the SCRBE approach presented is not limited to bearings and the RLE, but directly extends to other component-based systems. We show numerical results for plain bearings to demonstrate the validity of the proposed approach.

AMS subject classifications: 76D08, 35J25, 65N30, 65N55

Key words: Reynolds lubrication equation, static condensation, domain decomposition, model order reduction, reduced basis element method, *a posteriori* error estimation.

1 Introduction

The Static Condensation Reduced Basis Element (SCRBE) method was recently introduced in [12] as a computational tool to efficiently analyze parametrized large-scale component-based structures. Such structures — which are composed of a large number of similar or identical parametrized components — naturally appear in many engineering applications. A building, for example, is composed of components like rooms, walls,

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hallways, and staircases; and each component may be described through parameters like geometry, material constants, and boundary conditions.

The SCRBE method combines two essential ingredients: non-overlapping domain decomposition (resp. substructuring) methods and reduced basis methods. The idea is to employ static condensation to eliminate the internal (to each subdomain resp. component) degrees of freedom in terms of the corresponding boundary or interface degrees of freedom. Evaluating the entries of the associated Schur Complement System, however, requires numerous evaluations on the subdomain, i.e., bubble solves. If standard discretization techniques like finite elements are used to solve for the bubble functions, this step can be quite expensive — especially if one is interested in analyzing many different parameter combinations. This is were the reduced basis method comes into play.

The reduced basis method [9, 21, 22] is a model order reduction technique which allows efficient and reliable reduced order approximations for a large class of parametrized PDEs and is thus used to approximate the bubble functions. The offline-online computational decomposition allows to move expensive precomputations to the offline stage, the bubble solves are then performed efficiently online. Furthermore, rigorous and efficiently evaluable *a posteriori* bounds have been developed for the system-level error of the SCRBE approximation with respect to the underlying finite element approximation [12]. Within the last two years, the SCRBE method has been extended to also incorporate port reduction [6, 7] and has been successfully extended to treat various engineering problems [11, 13, 23].

We note that the SCRBE method comprises ideas from the Reduced Basis Element (RBE) method [17, 18] and the classical Component Mode Synthesis (CMS) [5, 10]. The RBE method employs the reduced basis method to approximate the bubble functions, but couples the components through a mortar-type procedure. The CMS employs a static condensation to “couple” the components, but uses an eigenmodal expansion to approximate the bubble functions. Indeed, the SCRBE method advantageously combines both approaches: the reduced basis treatment of bubble functions enables parametric variations of the components, whereas component coupling through static condensation enables the derivation of rigorous system-level *a posteriori* error bounds.

In this paper, we employ the SCRBE method to study the pressure distribution within a plain bearing governed by the Reynolds Lubrication equation (RLE). Our main contribution is to extend the SCRBE methodology introduced in [12] to consider a more general domain decomposition. More precisely, in [12] each component is allowed to have at most one neighbor on each port. This assumption excludes the typical wireframe approximation [4], where more than two components “meet” at a junction. Here, we consider a two-dimensional rectangular computational domain, i.e., an unfolded plain bearing, which is decomposed into small rectangular components forming a checkerboard pattern; the interface thus contains junctions where four components meet. The wireframe approximation has implications on the definition of the port degrees of freedom as well as on the *a posteriori* error bound. We show how to extend the work from [12] to this case in the sequel. Furthermore, our second contribution is to present an improved, i.e.,