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Simulation of Compressible Two-Phase Flows Using a Void Ratio Transport Equation

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Abstract. A compressible and multiphase flows solver has been developed for the study of liquid/gas flows involving shock waves and strong expansion waves leading to cavitation. This solver has a structure similar to those of the one-fluid Euler solvers, differing from them by the presence of a void ratio transport-equation. The model and the system of equations to be simulated are presented. Results are displayed for shock and expansion tube problems, shock-bubble interaction and underwater explosion. Close agreement with reference solutions, obtained from explicit finite volume approaches, is demonstrated. Different numerical methods are additionally displayed to provide comparable and improved computational efficiency to the model and the system of equations. The overall procedure is therefore very well suited for use in general two-phase fluid flow simulations.

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Key words: Compressible two-phase flows, cavitation, homogeneous model, shock and expansion waves, inviscid simulation.

1 Introduction

Theoretical and numerical modeling of two-phase fluid flow problems is of practical importance in various areas of industry such as thermal power generation plants, propulsion, hydraulic turbines and also environmental applications. Despite their relevance in industrial and environmental applications, compressible two-phase flow investigations have remained complex and challenging areas of applied mathematics and computational methods. The most widely used modeling approach is based on averaged two-phase fluid flow model such as the one-fluid formulation. Within such averaged model,

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there are different approaches according to the physical assumptions of interest made on the local mechanical and thermodynamical equilibrium and to the slip condition between phases. This has resulted in the development of diverse models and system of equations ranging from seven to three equations only. The two-fluid approach is the most complete as it represents the dynamics of the quantities mass, momentum, energy and void fraction. One of the most used two-fluid model is the one proposed by Baer and Nunziato [1]. The computation of the two-fluid model is also known to be a real challenge [2–5]. As an alternative way, a reduced five-equation model has been derived with the assumptions of velocity and pressure equilibrium [6–8] and applied with success in various flows. By assuming the thermal equilibrium between phases, a four-equation model can be expressed. It is composed by three conservation laws for mixture quantities completed by an equation for a non-conservative quantity describing the flow topology, usually the void ratio [9, 10]. The source term includes explicitly the mass transfer between phases. A very popular formulation has been developed to simulate cavitating flows and involves tunable parameters for the vaporization and condensation processes (different sets of parameters are presented in [11]). The domain of variation of these coefficients is huge (from 1 to 10^6) leading to serious problems of calibration for different configurations. Moreover, this family of models is not thermodynamically well-posed and does not respect thermodynamic constraints [12]. With the assumption of complete thermodynamic equilibrium between phases (local temperature, pressure and free Gibbs enthalpy equality between phases), the three-equation models or Homogeneous Equilibrium Models (HEM) are derived [13–15].

A critical aspect for two-phase simulations concerns the numerical methods of interest and their accuracy problems. The hyperbolic nature of such flows and their characteristic analysis makes the simulation very stiff and challenging. Large variations of the speed of sound in the mixture is also a difficult problem. Indeed, the speed of sound can be several orders of magnitude higher in the liquid phase than in the two-phase mixture. The non-monotonic behavior of the sound speed in the mixture causes inaccuracies in wave's transmission across interfaces. In addition to that, the volume fraction variation across acoustic waves causes difficulties for the Riemann problem resolution particularly in the derivation of approximate Riemann solvers. This is due to the occurrence of the large discontinuities of thermodynamic variables and equations of state involved at material interfaces. As a result, numerical instabilities and spurious oscillations appear through the complete wave structure [16]. The reason for such unusual behavior lies in the numerical dissipation of the methods which reproduce a thermodynamic path that is not correct. This also implies computational failure for Godunov methods which is due to the large decrease of the pressure up to vacuum ghost.

In the present paper, a four-equation model is considered. The set of equations includes three conservation laws for mixture quantities along with a void ratio transport-equation [10, 17]. The mass transfer between phases appearing explicitly in the formulation is closed by assuming its proportionality with the mixture velocity divergence. This model does not involve any tunable parameters and the thermodynamic coherence is re-