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A Multiscale Algorithm for Heat Conduction-Radiation Problems in Porous Materials with Quasi-Periodic Structures

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Abstract. This paper develops a second-order multiscale asymptotic analysis and numerical algorithms for predicting heat transfer performance of porous materials with quasi-periodic structures. In these porous materials, they have periodic configurations and associated coefficients are dependent on the macro-location. Also, radiation effect at microscale has an important influence on the macroscopic temperature fields, which is our particular interest in this study. The characteristic of the coupled multiscale model between macroscopic scale and microscopic scale owing to quasi-periodic structures is given at first. Then, the second-order multiscale formulas for solving temperature fields of the nonlinear problems are constructed, and associated explicit convergence rates are obtained on some regularity hypothesis. Finally, the corresponding finite element algorithms based on multiscale methods are brought forward and some numerical results are given in detail. Numerical examples including different coefficients are given to illustrate the efficiency and stability of the computational strategy. They show that the expansions to the second terms are necessary to obtain the thermal behavior precisely, and the local and global oscillations of the temperature fields are dependent on the microscopic and macroscopic part of the coefficients respectively.

AMS subject classifications: 35B27, 34E13, 74Q05, 83C30

Key words: Multiscale asymptotic analysis, radiation boundary condition, quasi-periodic structures, nonlinear heat transfer problems.

1 Introduction

Porous materials are widely applied in the aeronautic and aerospace engineering owing to the good thermal stability, low relative density and high heat resistance. In particu-

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lar, porous materials are usually designed for thermal protection system (TPS) during the spacecraft's flying out or re-entry into the atmosphere. Under such condition, the temperature of spacecraft's surface will be high enough in a moment, and heat radiation should not be omitted in actual calculations. Also, as the materials often have periodic configurations and characteristic coefficients oscillate sharply in small cells, it is necessary and important to develop an effective method to predict the thermal and mechanical performance of the porous materials.

Solving the heat radiative problem in porous materials by a direct numerical method becomes rather difficult since it would cost huge computer memories and time to accurately catch the local fluctuation behavior of temperature fields even for the supercomputer. Generally, homogenization methods and associated multiscale algorithms describe the global behavior by reduce the governing equations with rapidly varying coefficients to the equations with effective coefficients, which can not only save the computing resources but also guarantee the calculation precision [1–7]. Moreover, by adding appropriate correctors, the approximate solutions with oscillatory behavior can also be reproduced availably [8–10]. Up to now, some homogenization and multiscale methods were developed to study the heat conduction-radiation problems arising from porous materials. Liu and Zhang [11] investigated the effective macroscopic properties of radiative-conductive heat transfer problems in periodic porous materials. Bakhvalov [12] gave the asymptotic expansion forms for the solutions of those problems. Allaire and El Ganaoui [13] studied the heat transfer problems with ε^{-1}-order radiation boundary by two-scale expansion methods, and justified the convergence. Meanwhile, Ma and Cui [14] proposed a second-order two-scale method to solve the coupled problems, and obtained the convergence order with $\mathcal{O}(\varepsilon^{1/2})$. Cui et al. [15–17] discussed the heat conduction and radiation problems in periodic or random porous materials, and developed a newly higher-order multiscale method for the problems. Later, Yang, Sun and Cui [18, 19] obtained the second-order multiscale solutions for the dynamic thermo-elastic problems of porous materials with interior surface radiation. Obviously, from the works mentioned previously, the homogenization method and the associated multiscale techniques can give sufficiently effective predictions of the thermal and thermo-mechanical coupling properties of arbitrarily sophisticated microstructures. Also, such techniques and algorithms can perform calculation of the temperature and heat flux fields on the macro scale according to the effective coefficients obtained at the microscopic scale.

Generally speaking, owing to composite materials manufacturing technology, such as fatigue damages, the material parameters are no longer periodic, but local-periodic, i.e., quasi-periodic. In other words, the material coefficients can depend not only on the microscale information but also on the macro location. Functionally gradient materials (FGM) are a representative material with quasi-periodic structures [20–23]. Based on the traditional homogenization methods, Lions [1] systematically investigated the elliptical boundary value problems with quasi-periodic structures. Wirth [24] proposed a space dimensions method multiplied by different scales to efficiently treat quasi-periodic and multiscale problems. Andrianov et al. [25] applied a novel asymptotical approach to in-