

## The Nonlinear Dynamic Behaviors of Electrostatic Micro-Tweezers

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**Abstract.** This work addresses the nonlinear dynamic behavior of different electrostatic micro-tweezers, a micro electric actuator. This actuator, a cantilever beam electrostatic micro-tweezers, has been extensively used in micro-electro-mechanical systems (MEMS). The importance of micro electric actuators manufactured is higher than the other part of MEMS since it is the power source of the entire micro-electro-mechanical systems. In actual operation, the instability and bad dynamic characteristics of the electric actuators will cause larger displacement mobility error, such as transport behavior and response procedures failure, etc., and even damage the micro-electro-mechanical systems. To improve the actuator dynamic displacement accuracy, the dynamic behavior in the electric actuator system must be studied, especially for nonlinear dynamic behavior of system. In this work, the differential quadrature method (DQM) was employed to solve the problem of nonlinearity in the equation of motion. The results reveal that the proposed DQM model can be used to simulate the nonlinear behavior of the micro-tweezers efficiently. Micro-tweezers of various shapes were studied to examine the feasibility of applying the DQM in analyzing their nonlinear responses. The simulated results agree very closely with the calculated and experimental data in the literature.

**AMS subject classifications:** 35G50, 37M05, 65E05

**Key words:** Microelectromechanical, pull-in, Differential Quadrature Method (DQM), MEMS, electrostatic, micro-tweezers.

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## 1 Introduction

Cantilever beam electrostatic micro-tweezers have been extensively used in microelectromechanical systems (MEMS). Some investigations [1, 2] developed and simulated

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the chemical vapor deposition (CVD) of tungsten with isolated silicon dioxide to form the microelectromechanical structures of micro-tweezers. In [3], a combination exterior boundary-element method (BEM) for determining the electrostatic force and a finite-element method (FEM) for determining the elasticity of beam were proposed to quantify the coupling between the electrostatic force and the elastic deformation of the micro-tweezers.

Some three-dimensional methods have been developed for evaluating the nonlinear behavior of electrostatic devices. A 3-D structural simulator, OYSTER, was proposed in [5] to address 3-D geometric effects in the fabricated process. The importance of the electrostatic effect in the design and modeling of micro-actuators was discussed in [6]. Senturia et al. [7,8] developed MEMS systems using the ABAQUS package and the BEM program, FASTCAP, for electrostatic analysis to solve the nonlinear-coupled equations.

The pull-in behavior of different beam structures and pressure sensors [9] was elucidated using three models, i.e. the lumped parallel-plate spring model, the one-dimensional analytical model and the three-dimensional finite-element model. In [10], the pull-in instability phenomena of different beam actuators was used to determine the corresponding MEMS material parameters.

In 1997, Legtenberg et al. [11] employed the Rayleigh-Ritz method to examine the large displacement characteristics of cantilever beam type actuators. In [12], the effect of squeeze-film damping on the dynamics responses of a nonlinear actuator was studied using three-dimensional MEMCAD and FEM programs in [12]. In [13], leveraged bending and strain-stiffening methods were presented to enlarge the distance traveled before the electrostatic actuator is pulled in. The SOI (silicon-on-insulator) was used to fabricate the micro-actuators and micro-grippers, which provide the advantages of being stable and requiring a few fabrication steps [14]. In [15], SU8 photoepoxy with its excellent aspect ratio and attainable film thickness was used to fabricate shape memory alloy and actuated micro-grippers. Researchers [16, 17] focus the new applications of micro grippers on biological and micro robot. These studies developed to show the feasibility of micro robots and biological science. Recently, the electro thermal micro gripper was attracted to some investigations [18, 19].

In 1972, Bellman et al. [20] first proposed the differential quadrature method (DQM) to solve nonlinear partial equations. Before then, DQM had been widely employed to calculate the flexural vibration of a geometrically nonlinear beam [21], and to perform static and free vibration analyses of beams and plates [22]. Bert and Malik [23] used the differential quadrature method to evaluate the dynamic characteristics of various tapered rectangular plates. Bert [24] further applied particular boundary conditions to the differential quadrature weighting coefficient matrix, without applying the so-called  $\delta$  technique. Chen and Zhong [25] introduced Hadamard product to derive the weighting matrices for differential quadrature and differential cubature solutions to nonlinear differential and integro-differential equations. The Hadamard and SJT product approach was proposed in reference [26] to simplify the nonlinear equations and minimize the effort of evaluating the Jacobian derivative matrix by the Newton-Raphson method. The