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A Mean Curvature Regularized Based Model for Demodulating Phase Maps from Fringe Patterns

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Abstract. We introduce a variational method for demodulating phase maps from fringe patterns. This new method is based on the mean curvature of the level sets of the phase surface that is used for regularization. The performance of the method is illustrated with both synthetic and real data.

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1 Introduction

Fringe analysis techniques are very popular to estimate with reasonable accuracy physical quantities such as shape of objects, deformation, refractive index and temperature fields. They achieve these goals by recovering the local phase from one or a collection of interference fringe pattern images. The mathematical model of a fringe pattern is described by the equation

$$u = a + b\cos(\psi + \phi), \tag{1.1}$$

where a is the background illumination, b is the amplitude modulation, ψ is the spatial carrier frequency and ϕ is the phase map to be recovered. The problem of recovering not only ϕ but also a and b from the above equation is an ill-posed problem. Having only Eq. (1.1) to recover the three unknowns a, b and ϕ, plus the nonlinearity of the cosine function makes the problem very challenging. Recently, variational techniques that aim to reduce uncertainty of the solution by introducing more information into the model

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by means of regularization of the unknown variables have proved to deliver a feasible solution to this problem, see [10, 17, 30] and references therein. The new information introduced in the form of a regularizer defines the properties of the variational solution so a careful selection is advised.

The paper is organized as follows: in Section 2 we review the TV model, its virtues and drawbacks. In Section 3 we introduce a new curvature based model. In Section 4 we present the numerical solution of the Euler-Lagrange equations. In Section 5 are the experimental results and Section 6 is used to present our conclusions. In the Appendix at the end we present the derivation of the Euler-Lagrange equations.

2 Review

The use of regularization to process phase maps can be traced back to the works of [9, 13, 18, 25] and references therein. In those works some sort of smooth regularization was used with the purpose of stabilizing the solution of the proposed algorithms. However, it is not until very recently that cutting-edge variational techniques, already successfully proved in the field of image processing, have started to being applied to the modeling of interferometry problems. For instance, the very popular Total Variation (TV) regularizer was used in [19] for measurement of planar refractive index profiles with rapid variations in glass using interferometry. Likewise, in [22] the authors proposed a TV based single-shot interferogram analysis for accurate reconstruction of step phase objects.

A work of particular interest to us is the one we presented in [17], where a variational method was proposed for recovering a discontinuous phase map from a single pattern. This method, applied TV regularization to all three unknowns ϕ, a and b as it is shown below

$$\begin{aligned}\underset{a,b,\phi}{\operatorname{argmin}}\, TV(a,b,\phi,g)\equiv\Big\{\int_\Omega(u-g)^2 d\Omega+\lambda_1\int_\Omega|\nabla a|d\Omega \\ +\lambda_2\int_\Omega|\nabla b|d\Omega+\lambda_3\int_\Omega|\nabla\phi|d\Omega\Big\},\end{aligned} \tag{2.1}$$

where $\Omega\subseteq\mathcal{R}^2$ is the domain of integration, g is a given fringe pattern and $\lambda_i>0$, $i=1,2,3$ are regularization parameters.

The distinctive feature of (2.1) is that it allows the recovering of sharp phase transitions, something that other methods, such as those based on L_2 regularization, fail to deliver. However, recent studies [4, 24, 29] have shown that TV regularization brings some unwanted inconveniences as well. The most known being staircasing and the loss of signal height.

The contribution of this paper is to present a different regularizer, based on the mean curvature of the level sets of ϕ, which overcomes these drawbacks while still retaining the nice properties of the TV model.