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Two-Dimensional Multi-Domain Hybrid Spectral-WENO Methods for Conservation Laws

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Abstract. The multi-domain hybrid Spectral-WENO method (Hybrid) is introduced for the numerical solution of two-dimensional nonlinear hyperbolic systems in a Cartesian physical domain which is partitioned into a grid of rectangular subdomains. The main idea of the Hybrid scheme is to conjugate the spectral and WENO methods for solving problems with shock or high gradients such that the scheme adapts its solver spatially and temporally depending on the smoothness of the solution in a given subdomain. Built as a multi-domain method, an adaptive algorithm is used to keep the solutions parts exhibiting high gradients and discontinuities always inside WENO subdomains while the smooth parts of the solution are kept inside spectral ones, avoiding oscillations related to the well-known Gibbs phenomenon and increasing the numerical efficiency of the overall scheme. A higher order version of the multi-resolution analysis proposed by Harten is used to determine the smoothness of the solution in each subdomain. We also discuss interface conditions for the two-dimensional problem and the switching procedure between WENO and spectral subdomains. The Hybrid method is applied to the two-dimensional Shock-Vortex Interaction and the Richtmyer-Meshkov Instability (RMI) problems.

Key words: Spectral; WENO; multi-resolution; multi-domain; hybrid; conservation laws.

1 Introduction

In this article we extend the one-dimensional Multi-Domain Hybrid Spectral-WENO Method (Hybrid) for Hyperbolic Conservation Laws [5] to two dimensions in space and

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apply it to the classical Shock-Vortex interaction and Richtmyer-Meshkov instabilities problems. The general idea of the Hybrid method is to use a multi-domain framework in order to apply convenient spatial discretizations to the smooth and rough parts of the numerical solution. Shocks and high gradients are kept at WENO subdomains, while complex, but still smooth, details of the solution are treated within spectral subdomains. Numerical efficiency is increased with respect to the classical spectral and WENO methods: Postprocessing techniques of the spectral method approach of shocks [11, 16] are avoided, since no Gibbs phenomenon will occur, and the expensive characteristic decompositions and projections of the WENO method are skipped at the smooth parts of the solution [2, 3, 6, 9, 10, 15].

The main issues in the construction of the Hybrid method are the smoothness measurement of the solution and the subdomains types switching algorithm. In this work we employ the high order multi-resolution algorithm by Ami Harten [12] to build a local classification of the solution into smooth and rough. Originally built to decrease the work of the fluxes computations of Conservation Laws, Harten's Algorithm proposes to use information from coarser grids when the solution is locally over-represented. We instead use the multi-resolution information to apply distinct numerical methodologies to the different structures of the solution. The main goal is to conjugate the higher efficiency of the spectral method with the shock-capturing capability of the WENO method. The multi-resolution analysis is used to trigger the switching algorithm to change the subdomains spatial discretizations if shocks start to develop at a spectral subdomain, or if the solution becomes smooth at a WENO one. Moving discontinuities are similarly treated by changing to (or maintaining as) WENO the subdomains on their paths and switching to (or maintaining as) spectral the subdomains that were left behind. These changes are performed via Lagrangian and spectral interpolations of the local solutions to the new discretizations grids. Interpolation is also used to patch the solutions at the interfaces. While a simple average is sufficient for the interfaces where the solution is smooth, using the same grid spacing at adjacent WENO subdomains is necessary for a conservative transmission of shocks [24]. Even though we do not have a theoretical proof of the conservation of the Hybrid scheme, we argue that with the conjugation of two conservative schemes with a conservative WENO interface and the high-order accuracy of the conservative spectral scheme, the conservation error should be spectrally small. We have numerically demonstrated this fact in [5] through a long-time integration of the inviscid Burgers equations with correct shock speed and achieved excellent agreement with the analytical solutions of the standard Riemann shock-tube problems, such as the Lax and the Sod problems of the Euler Equations.

The paper is organized as follows: Section 2 provides quick reviews on spectral and WENO methods. The multi-resolution analysis is discussed in details at Section 3 and the Hybrid Method is introduced at Section 4. The Switching Algorithm is presented in Section 4.3 and numerical experiments with two dimensional compressible flows are finally presented at Section 5. Concluding remarks are given in Section 6.